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Decomposing the VIX Index into Greed and Fear

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Abstract

Greed and fear are the main psychological factors driving investment decisions, and the VIX Index is regarded as the most important measure of how fearful the market feels about future returns of the main equity index, the S&P 500 Index. However, given that the VIX is calculated by combining both upside expected volatility implicit in out-of-the-money calls and downside expected volatility implicit in the value of out-of-the-money puts, the taken-for-granted assumption that a rising VIX should be interpreted as a sign of growing fear in the equities market can be misleading. In this paper we formally deconstruct the index into two components, the upside and the downside expected volatility, in a similar fashion as it is done in statistics with the semi-variance. We then propose a Greed-Fear index using the data obtained to provide a better gauge about investors' sentiment on the market.

Keywords — VIX, Volatility, Greed-Fear index, Variance Swap

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1 Introduction

Finance is about managing risks, and risk is a way to approximate uncertainty. Most valuation models in finance aim at appropriately measuring risk and evaluating the expected return on spot fair price that compensates for it, allowing investors to sell risk when they consider it expensive (i.e., expected returns do not compensate for it) and to buy it when it is supposedly cheap (expected returns overcompensate for risk).

However, the million-dollar question is: How do we know whether an asset is riskier than other? Though there are myriad of metrics in the market, a simple way to capture risk is by calculating the standard deviation of returns.

Considering the importance of this concept in finance, and the fact that options capture risk in their price, in 1993 the Chicago Board Options Exchange (CBOE) introduced the Volatility Index (VIX Index), which became one of the most popular US Capital Markets indicator, labeled as an "uncertainty index" or "fear gauge". Despite the label and the fact that increasing volatility could point to increasing uncertainty in the market, the index actually measures the implied expected volatility of returns (capturing the implied 30-day volatility of the S&P 500 Index through out-of-the-money (OTM) option prices), combining upside and downside volatility (as in any calculation of variance or standard deviation).[4]

From our point of view, upside expected volatility could be understood differently from downside expected volatility, as it is done with the concept of semi-variance (or its square root, the semi standard deviation) of realized returns when calculating downside risk in portfolio management, provided that returns distribution is not symmetric (skewed) and perhaps with fat-tailed. In this sense, a rising VIX could be indicative of greedy investors pushing call option prices up, and not fearful investors selling or hedging with put options.

Moreover, it is the purpose of the present paper to formally decompose the VIX into an upside expected (implied) volatility, measured by OTM call options, and a downside volatility, measured by OTM put options. In the former case, the fact that stocks can further go up should not be considered as fear, but eventually as greed. In the latter case, the price of insurance against a downturn could effectively signal a higher degree of awareness about future negative price movements, and hence increased fear.

The structure is the following: section I gives a formal introduction of the model and reviews past literature contributions to the topic; section II describes thoroughly the implemented methodology; section III exposes the obtained results; and finally,

[4] For a comprehensive study on VIX Futures, see, [Avellaneda and Papanicolaou, 2018].

section IV discusses the main conclusions and further developments.

2 Review and Past Literature

Greed and fear, when considered in financial markets psychology, are viewed as opposing states, causing increased uncertainty and deviations from the Efficient Market Hypothesis (EMH).[5] These inconsistencies are associated with investors irrational behavior when making financial decisions. Greed and fear are also among the animal spirits that Keynes identified as profoundly affecting economic and financial decision making. Given the fact that investors give a great deal of importance assessing the current state of the markets, several indicators have been developed. For example, CNN launched an index that considers seven factors to discern how fearful and greedy market participants are, rating investor sentiment on a scale of 0 to 100. One of these indicators is actually estimated from the VIX Index.[6]

The fact that the VIX Index could be understood as a fear index is subscribed by Carr (2017), where he suggests four reasons supporting the concept of VIX as a "fear gauge", namely: assuming that risk-averse investors are long the S&P500, increases in expected variance or risk aversion raise VIX and lower expected utility; the argument that VIX has the same value as the path-independent log-contract at T and that the latter claim has negative delta; the leverage effect implies that abnormally high VIX levels tend to be accompanied by abnormally low S&P500 levels; and finally, when the Black Scholes model is holding, the VIX construction has more dollars invested in ATM and OTM puts than in ATM and OTM calls, so if the ATM and OTM puts reflect fear while the ATM and OTM calls reflect greed, then VIX is more about fear than greed. Furthermore, when the negative skew is taken into account, this fear to greed ratio increases. They conclude that there are valid reasons for regarding VIX as both a volatility index and a fear gauge. As we see, the point that calls tend to reflect greed and puts fear is mentioned by the authors.

In an article herein mentioned, a veteran options trade claims: "It is not now, nor has it ever been, the fear index. It was constructed to be the best estimate that we can come up with for 30-day volatility in the SP 500." To strength the point, the article mentions that the current level of the VIX — around 20, historically interpreted as showing a higher level of "fear", and at odds with that rally in the major averages. The answer suggested in the article is that speculative call buying has soared in that period.[7]

5 See, e.g., [Fama and Eugene, 1970].

6 https://money.cnn.com/data/fear-and-greed/

7 https://finance.yahoo.com/news/stop-calling-the-vix-the-fear-index-veteran-options-trader-205914937.html

The point is key to our work, the fact that variance averages the square of positive and negatives deviations of the observations with respect to the mean= and that the VIX calculation follows the same concept, by mixing upside and downside expected movements of returns.

In this vein, a well-established risk metric used by financial practitioners is the semi-variance, i.e., the variance which only considers observations that are below the mean, being a valuable metric as it measures the downside risk, one of the biggest risk factors for long-only big institutional investors.[8]

In the next section we are going to deconstruct the VIX into the two semi-variances or semi standard deviations, acknowledging the upside expected volatility as a symptom of greed, and the downside expected volatility as a symptom of fear.

3 Model and Methodology

Volatility can be measured through a variety of approaches. The most common is the sample standard deviation estimator, usually called "realized volatility" or "historical volatility (HV)". Other approaches have been proposed, with the aim of capturing some stylized facts observed in financial time series, such as volatility clustering and volatility leverage. In this line, GARCH-type models have been widely used. However, as with everything in finance, they are not a panacea as they have several shortcomings.

To overcome some of the more controversial shortcomings and assumptions of time series-based volatility estimators, the investment community often chooses to infer this quantity from option prices. Since option prices are readily available for most liquid markets, they can be used to infer the volatility of the underlying. This metric, inferred from market prices, is usually called forward-looking "implied volatility" (IV).

VIX index values are calculated using the Cboe-traded standard SPX options. The time-to-maturity of the options selected to the volatility estimation ranges from 23 days to 37. Theoretically, the VIX index works as follows: it estimates the expected volatility of the S&P 500 index by aggregating the weighted prices of multiple SPX puts and calls over a wide range of strike prices. To qualify for the index calculations, the options should have non-zero bid and ask prices. For detailed calculations with example, one can refer to the section "VIX Index Calculation: Step-by-Step" of the VIX white-paper.

[8] See, for example, [Roy, 1952].

3.1 Derivation of VIX formula

The new VIX model-free formula relaxes some assumptions made by models like Black-Scholes. The strongest assumption here is about the underlying diffusion process, which is assumed to be a jumpless geometric Brownian motion.

Let's define a (filtered) probability space $(\Omega, \mathcal{A}, \mathbb{F}, \mathcal{P})$ with a complete and right-continuous filtration, where Ω is the sample space, $\mathcal{A}$ is a σ-algebra and $\mathcal{P}$ is the probability measure mapping $\mathcal{P} : \mathcal{A} \to [0, 1]$, equipped with the filtration $\mathbb{F} := (\mathcal{F}_i)_{i \in I}$ where $\mathcal{F}_i$ is a sub-σ-algebra of $\mathcal{A}$ for every $i \in I$. Let us define the underlying price process on this space

$$\frac{dS_t}{S_t} = rdt + \sigma_t dW_t \tag{1}$$

where r and σ is the risk-free rate and the instantaneous volatility function, respectively, and W is a Wiener process on the same probability space. In order to calculate the VIX index, we need the forward price F at time T under the risk-neutral probability measure Q, such that $F = \mathbb{E}^Q[S_T]$

$$\mathbb{E}^Q[S_T] = S_0 \exp\left\{\left(r - \frac{\sigma^2}{2}\right)T + \frac{\sigma^2 T}{2}\right\} \tag{2}$$

which naturally leads to expected value of the variance

$$\mathbb{E}^Q[\sigma_t^2] = \frac{2}{T}\ln\left(\frac{F}{S_0}\right) - \frac{2}{T}\mathbb{E}^Q\left[\ln\left(\frac{S_T}{S_0}\right)\right] \tag{3}$$

Applying Ito's lemma, we integrate the above equation such that:

$$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[\int_0^T \frac{1}{S_t}dS_t - \ln\left(\frac{S_T}{S_0}\right)\right] \tag{4}$$

The above derivation is usually denoted as the continuously-sampled variance. Solving the expected value of Eq. 4 under the appropiate risk neutral measure, we have:

$$\hat{\sigma}_t^2 = \frac{2}{T}\mathbb{E}^Q\left[\int_0^T \frac{1}{S_t}dS_t - \ln\left(\frac{S_T}{S_0}\right)\right] \tag{5}$$

As the expectation of a driftless Ito's process is zero, we have that (setting $r = 0$):

$$\mathbb{E}^Q\left[\int_0^T \frac{1}{S_t}dS_t\right] = 0 \tag{6}$$

So, the Eq. 4 becomes:

$$\widehat{\sigma}_t^2 = -\frac{2}{T}\mathbb{E}^Q\left[\ln\left(\frac{S_T}{S_0}\right)\right] \tag{7}$$

As the expected total variance depends on the expected value of a log-contract, which is not actively traded, we need a replication strategy for every stock level at expiration. It can be divided into i) a linear component and ii) a non-linear component. The first component corresponds to a forward contract with maturity T while the second component can be replicated with (plain vanilla) options, assuming a continuum of strikes for call and puts with expiration T. So, we have:

$$\ln\frac{S_T}{K_0} = \underbrace{\frac{S_T - K_0}{K_0}}_{forward\ contract} - \underbrace{\int_0^{K_0}\frac{(K-S_T)^+}{k^2}dK}_{put\ options} - \underbrace{\int_{K_0}^{\infty}\frac{(S_T-K)^+}{k^2}dK}_{call\ options} \tag{8}$$

Following the risk-neutral measure, dS_t is a martingale. So, $\mathbb{E}^Q[S_T] = S_0 = K_0$. Using the fact that discounted stock prices are Q-martingales, the total variance equation reads

$$\begin{aligned}\widehat{\sigma}_t^2 = &\frac{2}{T}\left[\ln\left(\frac{F}{K_0}\right) - \left(\frac{F-K_0}{K_0}\right)\right]\\ &+ \frac{2}{T}e^{rT}\left[\int_0^{K_0}\frac{(K-S_T)^+}{k^2}dK + \int_{K_0}^{\infty}\frac{(S_T-K)^+}{k^2}dK\right]\end{aligned} \tag{9}$$

At this point, we have obtained the formula for the expected total variance over the interval [0, T]. The next step is the discretization. The first term in Eq. 9 can be expressed in terms of its Taylor polynomial of the $\ln(\cdot)$, i.e.,

$$\ln\left(\frac{F}{K_0}\right) = \left(\frac{F}{K_0} - 1\right) - \frac{1}{2}\left(\frac{F}{K_0} - 1\right)^2 + \mathcal{O}(\cdots) \tag{10}$$

rearranging and adding the remaining terms, we obain:

$$\frac{2}{T}\left[\ln\left(\frac{F}{K_0}\right) - \left(\frac{F-K_0}{K_0}\right)\right] \approx \frac{1}{T}\left[\frac{F-K_0}{K_0}\right]^2 \tag{11}$$

The above term is regarded to the adjustment required since K_0 is adopted instead of F. As for the second term in Eq. 9, we need to discretize the strike sum as there is no strikes continuum in real-world markets. We have n strike levels, so the ΔK is obtained as:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}, \ \forall \, i = 2, \ldots, n-1 \tag{12}$$

We define a function for the average value of put and call prices in the limit of $K_i = K_0$:

$$Q(K_i) = \begin{cases} c(K_i), & K_i > K_0 \\ p(K_i), & K_i < K_0 \\ \frac{c(K_i)+p(K_i)}{2}, & K_i = K_0 \end{cases} \tag{13}$$

where $c(K_i)$ and $p(K_i)$ are the average of the bid-ask price. So, the discretized formula for the total variance is given by

$$\sigma_t^2 = \frac{2}{T} \sum_{i}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \tag{14}$$

with the VIX index defined as $\sigma_t = 100 \cdot \sqrt{\sigma_t^2}$.

3.2 VIX decomposition: fear or greed?

In the present section, we propose the VIX Index decomposition into two components: i) the upside implied volatility using the total variance of call options, and ii) the downside implied volatility using the total variance of put options, which is related to the concept of semivariance.

$$\sigma_{call,t}^2 = \frac{2}{T} \sum_{i}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} c(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \tag{15}$$

$$\sigma_{put,t}^2 = \frac{2}{T} \sum_{i}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} p(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \tag{16}$$

4 Data and Empirical Test

Should write about our replication, how close it is to the CBOE VIX Index (we have differences as we don't have the number of strikes the CBOE has).

4.1 Greed and Fear: Empirical Test

The two indices should be discussed: call volatility and put volatility. Show the contribution of both relative to the aggregate VIX index (defined as $\frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$). We can also show the difference in the VIX in the near and next term. Can we compare to the Skew index?

Using daily options of the S&P 500 we reconstructed the VIX index and then, decomposed to study the inter-temporal contribution of Calls and Puts.

Figure 1 shows the replication of the VIX and its original counterpart. As shown, the replication is very close. The differences come from data availability, that is, the strikes available in our dataset are not the same as the data available in the CBOE dataset, which is giant. However, as expected, the options used did a decent job replicating the original VIX.

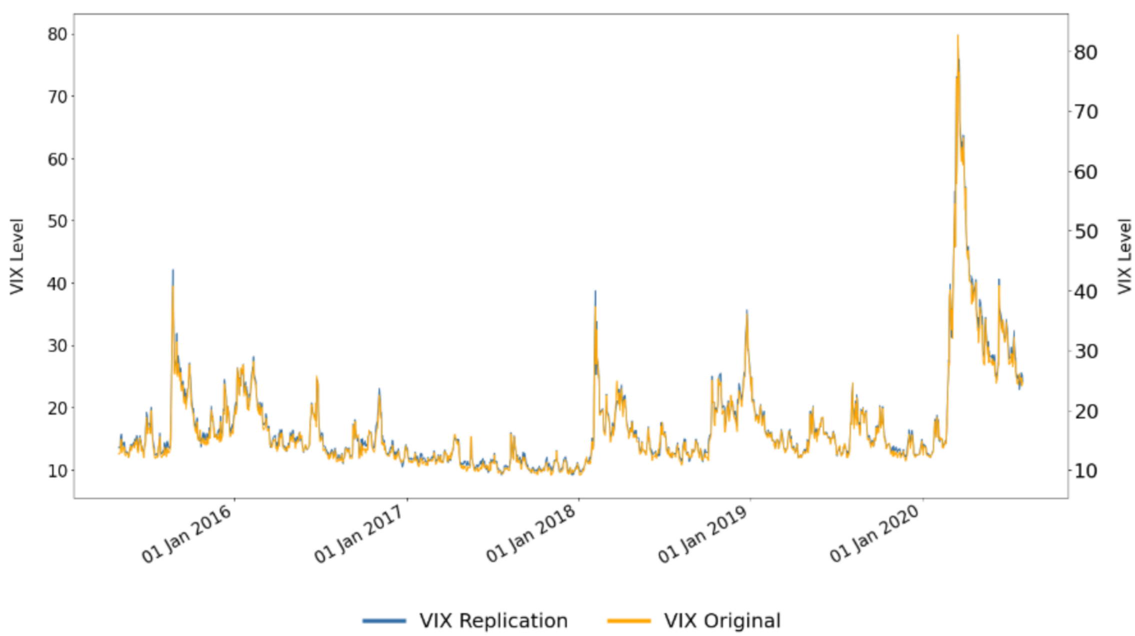

Figure 1: VIX Replication and Original VIX.

Figure 2 shows two indices. The "Put VIX" index and the "Call VIX" index.

The Put leg is most of the time higher than the Call leg, which is expected since empirically the Put options are more expensive than the Calls counterpart. This could be related to the Skew index, which should be analyzed in more detail to try to find signals.

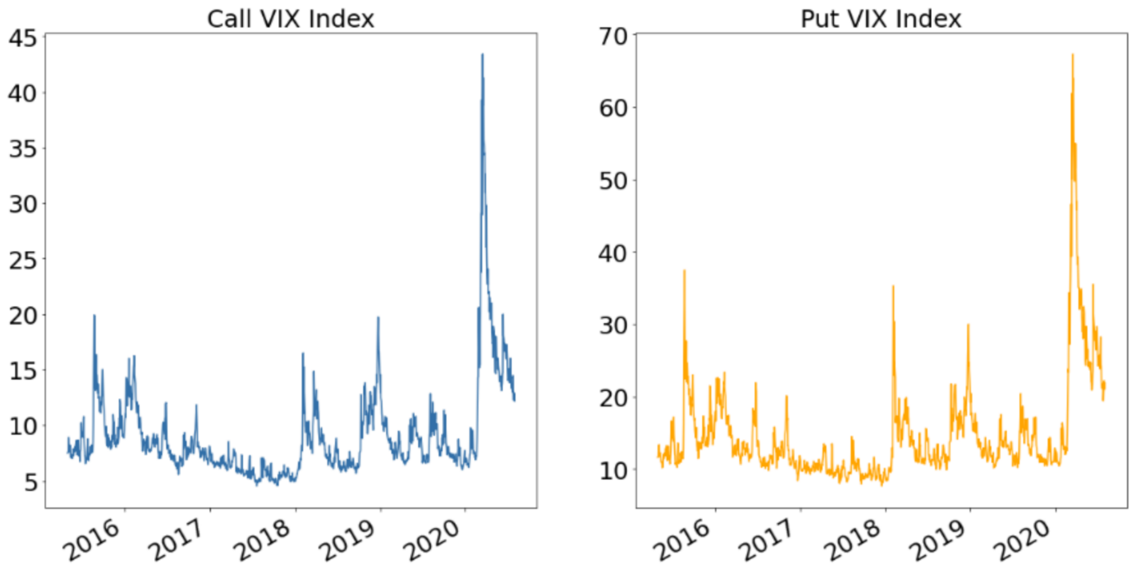

Figure 2: VIX decomposed into Calls and Puts. As expected, "Put VIX" is most of the time higher than the "Call VIX".

Figure 3 shows a very interesting finding related to the previous figure. The contribution of the Put leg to the VIX index is, most of the time, more than 2 times higher than the contribution of the Call leg, and this is amplified during market crashes. For example, during February 2018, during the so-called "Volmageddon", the ratio increased to about 5.

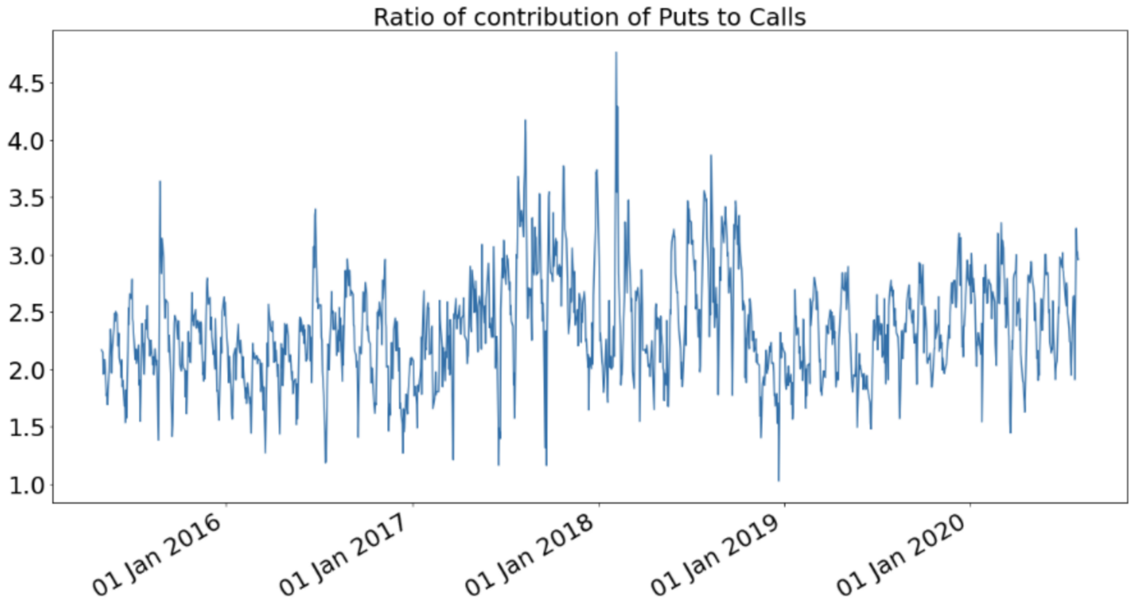

Figure 3: Ratio of Puts contribution to the Calls contribution.

Figure 4 shows the empirical PDFs of the actual VIX Index and the components studied above. As expected, the mean value of the Call VIX is the lowest and has the highest Kurtosis, while the Put VIX PDF ranges between the VIX Index PDF and the Call VIX PDF.

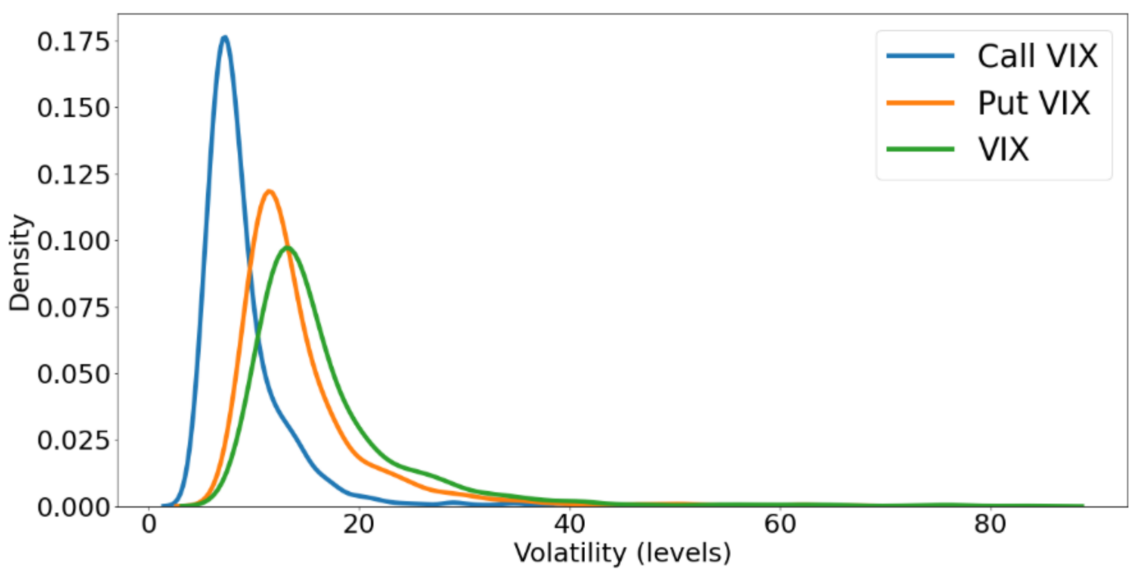

Figure 4: Empirical PDFs of the VIX Index and the Call/Put Components.

5 Conclusion and Further Research

The main purpose of this paper has been to show that the concept of semi-variance, widely used in finance as a proxy for downside risk, can be applied also to the VIX index.

Given that the VIX index is calculated by combining both upside expected volatility implicit in out-of-the-money calls and downside expected volatility implicit in the value of out-of-the-money puts, the index can be decomposed into two metrics, showing from our point of view two different things. The upside implicit expected volatility can be seen as a proxy of how greedy the market is, raising the value of calls and hence raising implicit volatility embedded in them; on the other hand, the downside expected implicit volatility can be seen as a proxy on how fearful the market is about the future, looking for protection.

Even though the put/call ratio can provide similar information, we acknowledge our decomposition as a better understanding of the underlying risk and sentiment.

We regard for future research the comparison between the information provided by these two ratios, and the understanding of what kind of information can help investor predict the real sentiment of the market and hence help them take better investing decisions.

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