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SOVEREIGN BOND SPREADS AND CREDIT SENSITIVITY

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Sovereign Bond Spreads and Credit Sensitivity

Ricardo Schefer [1]

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Abstract

Expectations of risky bond payments are unobservable and recovery rates for sovereigns are hard to estimate because they have no contractual claims to defined assets and samples of defaults are limited. A geometric version of credit spread is used to derive expected payments, dependent on idiosyncratic risk and unrelated to interest rates. The expectations are used to define a measure of price sensitivity to credit risk perceptions, or credit duration, improving the ambiguity of modified yield duration.

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Keywords: bond, sovereign, spread, expected, risk neutral, default, duration, yield

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Introduction

Most bonds are credit risky. The expected return of a credit risky bond is the relationship between its market value and the expectations of its promised cash flows. Expected payments are and recovery rates are not observable, but sets of implied cash flows can be derived from market values using a measure of credit spread dependent only on expected cash flows, unrelated to the interest rate.

The expectations of the promised cash flows payable at the promised dates are implied by the geometric risk premium on synthetic twin risk free bonds. Risk neutral expected payments are identified without subjective assumptions of recovery rates. This is important because sovereign bonds have no contractual or legal claims to defined assets, and also because the universe of defaulted issues is diverse and also too small to obtain good samples of recoveries.

Credit risk being the main attribute of this class of bonds, payment expectations are used to derive a measure of the sensitivity of risky bond price changes to changes in credit perceptions, or credit duration. This measure is more informative than modified yield duration because it is unambiguously related to expected payments, and independent of interest rate changes.

Spreads can define expected payments at the promised dates or may alternatively imply payment delays, or both. Following the general practice this paper mostly focuses on cash flows at promised dates.

The single period case

Risky bonds may be paid as promised, less than promised, later than promised, or a combination of both defaults. Expected payments are not observable, but a pattern may be implied by their risk premium.

The value of a future risky promised payment of C in one period is given by

$$V_R = C\,(1+y)^{-1} \qquad (1)$$

where y is the yield to maturity.

If investors are risk neutral, the value V_R can also be written as the present value of the expected payment E[C] of the promised payment C

$$V_R = E[C]\ (1+r)^{-1} \qquad E[C] < C \qquad (2)$$

where r is the rate of a same dated risk free zero coupon.

By equating (1) and (2) the geometric credit spread s defining y is found

$$\frac{E[C]}{C} = \frac{(1+r)}{(1+y)} = \frac{1}{1+s} \qquad s > 0 \qquad (3)$$

The credit spread s is

$$s = \frac{C}{E[C]} - 1 \qquad (4)$$

dependent solely on the promised payment and its expectation, independent of the risk free rate r, and different from the standard definition of spread s = y – r .

Thus, the credit spread reflects the ratio of the expected to promised payment ratio e:

$$e = \frac{E[C]}{C} \qquad (5)$$

The following table shows the relationship between e and the spread s:

Table 1 – The one period relationship between expected payment ratio e and the spread s

E	s
1.00	0.0%
0.95	5.3%
0.90	11.1%
0.85	17.6%
0.80	25.0%
0.75	33.3%
0.70	42.9%
0.65	53.8%
0.60	66.7%
0.55	81.8%
0.50	100.0%
…	...

For example, consider a promise to pay C= 100 at time 1 which is assessed to have an E[C] = 50, or an e = 0.5, like the result of a coin toss resulting in heads. Since, from (4) (5)

$$e = \frac{E[C]}{C} = \frac{1}{(1+s)} \qquad (6)$$

for r = 5%,

$V_R = \frac{E[C]}{1.05} = 47.62$, so that $y = \frac{100}{47.62} - 1 = 110\%$, and s = 100%, independent of the interest rate r[2].

In contrast, by using an arithmetic credit spread given by y – r = 105%, an observer would misidentify the implied expected payment. Furthermore, the arithmetic spread s = y – r covaries with the interest rate, which is unrelated to the expectation value.

An alternative interpretation of the value of a risky bond is that the promised payment may be assessed to be made at face value but at a date later than promised. This form of default can be written as

$$V_R = C\,(1+r)^{-H} \qquad H > T \qquad (7)$$

where H is the risk neutral expected time of payment C, later than T.

Equating (1) and (7),

$$(1+r)^{-H} = (1+y)^{-T} \qquad (8)$$

solving for H

$$H = T\,\frac{\ln(1+y)}{\ln(1+r)} \qquad r > 0 \qquad (9)$$

or

$$H = T\,\frac{\ln((1+r)(1/e))}{\ln(1+r)} \qquad (10)$$

For example, for r = 2%, a one year risky zero coupon with a face value of 100 and a market value of 88.24 may be expected to receive the promised payment C in 6.32 years, more than five

2 This is the case where investors form a payment expectation based on idiosyncratic attributes where the bond issuer's budget is unaffected by interest rates.

years later than promised, earning the risk free rate of return.

Therefore a risky bond's price may reflect either an expected a payment E[C] less than C at the promised date or the promised payment C at a later date.

This explains that investors may not be necessarily assuming a recovery cash flow at default time but rather an extension of the promised maturities of payments through a bond exchange, mutation, or debt restructuring. This is especially applicable to sovereign bonds which do not have defined contractual nor legal claims on the issuer's assets.

The multiperiod case: a T periods single payment

The present value of a future risky payment of C in T periods is given by

$$V_R = C\,(1+y)^{-T} \qquad (11)$$

where y is the yield to maturity.

Assuming risk neutrality

$$V_R = E[C]\,(1+r)^{-T} \qquad (12)$$

and the payment expectation is

$$e^T = \frac{E[C_T]}{C_T} \qquad 0 \le e < 1 \qquad (13)$$

so that

$$V_R = e^T\, C_T\,(1+r)^{-T}, \qquad (14)$$

and

$$e^T = (1+s)^{-T} \qquad s > 0 \qquad (15)$$

so that the expected payment of C at time T is

$$E[C_T] = C_T\,(1+s)^{-T} \qquad (16)$$

and the value of the risky bond is

$$V_R = C_T\,(1+s)^{-T}\,(1+r)^{-T} \qquad (17)$$

Mutiperiodic payments

The extension to multi period payment promises follows:

$$V_R = \sum_{t=1}^{T} C_t \, (1+r)^{-t} \, (1+s)^{-t} \quad [3] \qquad (18)$$

and the promised yield to maturity is

$$y = (1+r)\,(1+s) - 1 \qquad (19)$$

The geometric credit spread provides a straightforward interpretation of the yield to maturity y since

$$\frac{1}{(1+s)^t} = \frac{E[C_t]}{C_t} = e^t , \qquad (20)$$

where e is the risk neutral expected payment ratio. [4]

For example, consider a 10 year bullet bond with an annual coupon of 5. Assume a risk free rate of 2% and a constant annual payment ratio e = 0.90, which defines the credit spread at 11.1% per annum. The market price is 55.38, and the yield to maturity is 13.3%. The implied expected payments are shown under the E[PMT] column, which are obtained by multiplying the promised payments by their e^t.

Table 2 – A 10 year bond with constant e per year

t		e^t	E[PMT]	PV E[PMT]
1	5	0.90	4.50	4.41
2	5	0.81	4.05	3.89
3	5	0.73	3.65	3.43
4	5	0.66	3.28	3.03
5	5	0.59	2.95	2.67
6	5	0.53	2.66	2.36
7	5	0.48	2.39	2.08
8	5	0.43	2.15	1.84
9	5	0.39	1.94	1.62
10	105	0.35	36.61	30.03
Σ			64.18	55.38

[3] A constant term structure of spreads and interest rates is assumed

[4] e is not a probability

The present value of these expectations at the risk free rate are shown in the last column. They are equal to the promised payments discounted at the bond's yield to maturity, and add up to the market value of the bond. The column e^t shows how payment expectations decrease at a constant rate over time. Although 1 minus e^t may be interpreted as a default probability at time t, the fact that different term structures of default or possible default rates at different times can result in the same value of the bond render this interpretation as extreme. Instead, the relevant interpretation of the geometric credit spread is its implication of expected cash flows.

Changes in credit risk

The relevant price risk of a credit risky bond is its sensitivity to changes in expected payments. These can measured by the relative change in a risky bond price due to a change in expected payments, or "credit duration". Given the value of a risky bond

$$V_R = \Sigma\, C_t\, e^{t} \cdot (1+r)^{-t} \tag{21}$$

its price sensitivity to credit changes is

$$\frac{d\,V_R}{d\,e}\,\frac{1}{V_R} = \frac{\Sigma\, t\, C_t\, e^{t-1}\,(1+r)^{-t}}{V_R} \tag{22}$$

which can also be written as

$$\frac{d\,V_R}{d\,e}\,\frac{1}{V_R} = \frac{\Sigma\, t\, C_t}{(1+s)^{t-1} \cdot (1+r)^{t}} \cdot \frac{1}{V_R} \tag{23}$$

or

$$\frac{d\,V_R}{d\,e}\,\frac{1}{V_R} = \frac{\Sigma\, t\, C_t}{(1+y)^{t}} \cdot \frac{(1+s)}{V_R} \tag{24}$$

reordering, this is equal to duration times one plus spread

$$-\ \frac{D\,V_R}{d\,e}\,\frac{1}{V_R} = D \cdot (1+s) \tag{25}$$

Therefore credit duration is always higher than yield duration in absolute terms:

e duration = (1+s) D (26)

For the bond in Table 2, with a duration of 7.34 years, credit duration, or e duration is:

(1 + 0.111) x 7.34 = 8.15%

Alternatively, the following table shows its calculation using (22)

Table 3 – Calculation of credit duration: price sensitivity to changes in payment intensity

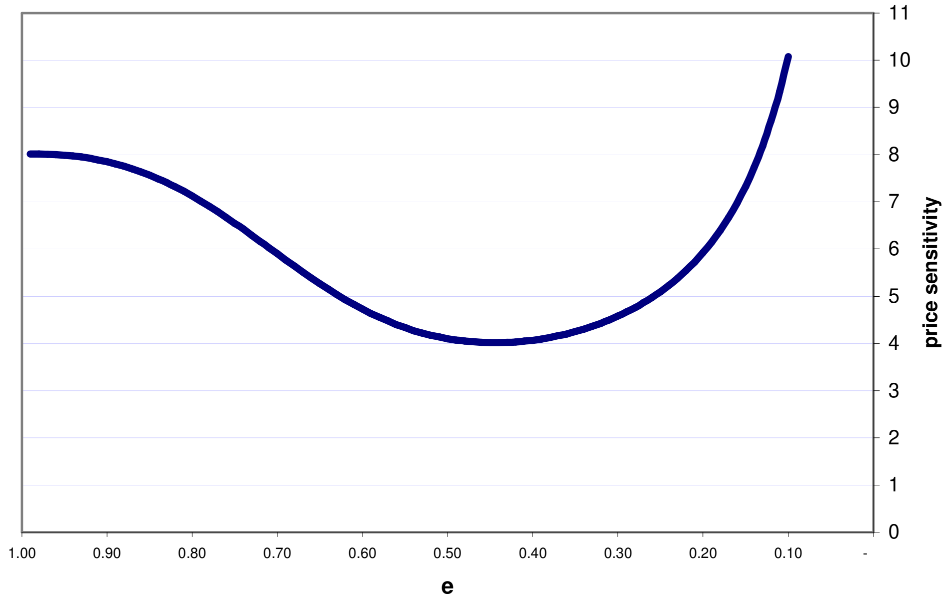
t	C_t	e^{t-1}	$(1+r)^{-t}$	derivative
1	5	1.00	0.98	4.90
2	5	0.90	0.96	8.65
3	5	0.81	0.94	11.45
4	5	0.73	0.92	13.47
5	5	0.66	0.91	14.86
6	5	0.59	0.89	15.73
7	5	0.53	0.87	16.19
8	5	0.48	0.85	16.33
9	5	0.43	0.84	16.21
10	105	0.39	0.82	333.71
			total	451.50
			total / V_R	**8.15**

The price relative change, or credit duration, to a 1 point drop in payment expectation from e= 0.90 to 0.89 is approximately -8.15%. This is larger in absolute terms than that estimated by modified yield duration for any e < 1 , the difference being greater the higher the credit risk.

Credit duration is unambiguous, while modified yield duration may reflect changes in yields due to either changes in the credit spread or to changes in the interest rate.

Contrary to modified duration, credit duration does not predictably fall as payment expectation e falls . For lower expected payment ratios the credit spread s rises, and offsets the decline in modified duration, as shown in the following graph depicting the 10 year bond in Table 2, for different values of e:

Graph 2 – Price sensitivity and e: a 10 year 5% coupon bond



Credit duration, like modified yield duration, assumes that expected payments will occur at the promised dates, or as a lump payment either in cash or with a mutated security at the duration date. If investors otherwise assume that a mutated schedule of payments will occur at dates after duration date, the sensitivity will be much higher.

Conclusion

Yield to maturities of sovereign credit risky bonds are determined by risk neutral geometric spreads. These identify a set of risk neutral payment expectations, independent from interest rates. For risky sovereign bonds subjective recovery rates, which are hard to estimate because of the absence of claims to defined assets, and by the lack of good samples, are thus redundant. Further, geometric spreads reflect payment expectation ratios that provide an unambiguous measure of price sensitivity to changes in credit perceptions, or credit duration, as compared to modified yield duration which may reflect either changes in spreads or in interest rates.

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