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Optimal Relevance in Imperfect Information Games

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To help incorporate natural language into economic theory, this paper does two things. First, the paper extends to imperfect information games an equilibrium concept developed for incomplete information games, so natural language can be formalized as a vehicle to convey information about actions as well as types. This equilibrium concept is specific to language games, because information is conveyed by the sender through the message's literal meaning. Second, the paper proposes an equilibrium refinement which selects the sender's most preferred equilibrium. The refinement captures the notion that the speaker seeks to improve its status quo, aiming at optimal relevance. Explicit coordination through verbal communication parallels the idea of implicit coordination through focal points.

JEL classification codes: D83, C72

Key words: cheap talk, signs, semantics, pragmatics, relevance, equilibrium selection

I. Introduction

I propose to analyze unilateral communication in imperfect information games with a framework where the sender may announce its intentions to the receiver using messages that have a literal meaning that both the sender and the receiver can understand. This complements earlier work on verbal communication in incomplete information games.

Vincent Crawford and Joel Sobel (1982) develop a game-theoretic representation of unilateral communication between a sender and a receiver in incomplete information games as *cheap talk*, where messages are payoff-irrelevant. As is typical in signaling games, the solution concept is based on Perfect Bayesian Equilibrium (PBE) where there is an

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assignment of signals (in this case, verbal messages) to each sender type in equilibrium.[1] However, the messages themselves are not essential. Joseph Farrell (1993: 515) points out that since meaning cannot be learned from introspection in cheap-talk models, any permutation of messages across meanings gives another equilibrium. This implies that if there is an informative equilibrium, there are infinitely many.

Building on the strategic framework in Crawford and Sobel (1982), and sparked by Farrell's (1993: 515) deep insight that, credible or not, natural language has a comprehensible meaning, Jorge Streb and Gustavo Torrens (2015) develop an alternative solution concept for language games that incorporates the literal meaning of the message as an additional source of information. They call the common understandings embodied in natural language *meaningful talk*. This is part of a semiotic-inferential process that restricts the beliefs that players may entertain in response to verbal communication. In a Meaningful-Talk Equilibrium (MTE), the assignment of messages to each sender type on the equilibrium path is based on (i) the literal meaning of the messages themselves and on (ii) whether the receiver trusts the messages or not.

This paper first extends meaningful-talk equilibria to imperfect information games where the sender announces its intended move. Since the sender has to pick both a message and a move, imperfect information games do not allow isolating information transmission. However, they allow formalizing an essential feature of natural language as a conventional sign that can point not only to types but also to moves.

This study is closely related to the analysis of imperfect information games in Roger Myerson (1989) and Stefano Demichelis and Jörgen Weibull (2008). Like Myerson (1989), we restrict the interpretation of messages in equilibrium. While Myerson (1989) uses natural language as the medium of communication, his setup and equilibrium concepts are quite different. The communication process is more structured since he has in mind a negotiation where there is a mediator between both parties; furthermore, the equilibrium concept is the Robert Aumann (1974) Correlated Equilibrium, since the mediator uses a correlating device to implement the equilibrium strategies.

[1] Cheap-talk models set language apart from other signals: while standard signals may be credible because choices are differentially costly, words have no direct payoff consequences so they are credible only if players share common interests (Robert Gibbons 1992: 210).

The meaning correspondence in Demichelis and Weibull (2008) — the relation between the announced message and the intended action — is close to our emphasis on literal meaning. They introduce lexicographic preferences of the sender for honesty to analyze meaning. Navin Kartik (2009) also points out that with costly talk a message has a literal or exogenous meaning in incomplete information games. However, costly talk overlooks the role of the receiver in a verbal communication process: without a common language, the speaker cannot verbally communicate meaning, either true or false, because there is no way for the hearer to understand the messages.[2]

Second, the paper proposes an equilibrium refinement for imperfect information games. In contrast to incomplete information games, the incentives of the sender are simpler to analyze because there is only one type of sender, so equilibrium selection is a matter of determining the sender's optimal strategy. This provides a possible formalization of the suggestion in Schelling (1960: 59) on explicit communication as a way to select among equilibria, which complements implicit coordination through focal points

The paper is structured as follows. Section II extends the Streb and Torrens (2015) meaningful-talk model. An equilibrium concept for imperfect information language games is defined in which linguistic signs themselves are the source of information. An equilibrium refinement is proposed which selects the sender's (weakly) preferred equilibrium. Section III illustrates the impact of verbal communication in several classic games. The application of the equilibrium refinement to select among equilibria links this semiotic-inferential approach to the notion of optimal relevance in Dan Sperber and Deirdre Wilson (1995). This might also be expressed in terms of different degrees of precision of the sender's messages (Sobel 2011). Section IV shows that this approach can be interpreted as a formal pragmatics where the equilibrium meaning depends on the specific strategic setup. Section V contains the closing remarks.

[2] This idea is key in semiotics. Roman Jakobson and Morris Halle (1956: 72), for instance, state that "the efficiency of a speech event demands the use of a common code by its participants." More generally, Sperber and Wilson (1995: 43) point out that communication is an asymmetric process where "it is left to the communicator to make correct assumptions about the codes and contextual information that the audience will have accessible and be likely to use in the comprehension process."

II. Imperfect Information Language Games

The meaningful-talk model in Streb and Torrens (2015) is extended from incomplete information to imperfect information games. An equilibrium refinement is proposed to capture the suggestion in Schelling (1960) on verbal communication as a means to select among equilibria.

A. Moves and Messages

The sender S has to pick an action $a^S \in \mathrm{A}^\mathrm{S}$ and the receiver R has to pick an action $a^R \in \mathrm{A}^\mathrm{R}$. A natural language shared by both players, denoted by "$\mathbb{M}$", allows talking about different partitions of the world at large $\mathbb{W}$, with statements "Q" that point to a subset $\mathrm{Q} \subset \mathbb{W}$. Quotes differentiate two planes: reality and language. A subset "M" $\subset$ "$\mathbb{M}$" suffices to communicate about the actual moves $\mathrm{W} \subset \mathbb{W}$ of the sender in the game, namely, $\mathrm{W} = \mathrm{A}^\mathrm{S}$.

Semiotics distinguishes between natural signs and conventional signs (Ricardo Crespo 2012). In contrast to natural signs like smoke, conventional signs like language depend on each community: while “Fire!” is used in English, “¡Fuego!” is used in Spanish. Signs as such do not provide evidence of anything, they simply point to types or actions.

Conventional signs, or symbols, of which linguistic signs are the most important subset, have three elements (Daniel Chandler 1994): (i) the *signifier* "m", what in economics is typically called the *message*, e.g., “Meet me at noon at the information booth in Grand Central Station.”; (ii) the *signified* $\widehat{m}$, what we think of when we read or hear the message — a distinction is drawn below between the *literal* and the *equilibrium meanings*; and (iii) the *referent* m, where both the *object* to which the proposition allegedly applies, e.g., the intended action of the sender, and the *truth-value* of the proposition are considered.

Here messages are sentences that can express propositions, where reference assignment in a concrete context is needed to yield a full proposition which is either true or false. Symbolic information is considered as the sole vehicle for information transmission, abstracting from body language like tone of voice which can convey additional information.

In asymmetric information games, the referent is unobservable from the receiver's vantage point, so the receiver uses the literal meaning conveyed by the message to ascertain the intended action. The sequence is as follows. First, the priors $p(a^S) \in \mathrm{P}$ about the possible moves $w \in \mathrm{W}$ are given by an equilibrium of the game without communication, which in most examples below is the most diffuse mixed strategy Nash equilibrium. Second, the sender S sends a message $"m" \in "\mathrm{M}"$ about $w \in \mathrm{W}$ if unilateral communication is possible. Third, the receiver R updates its priors through the decoding and inferential steps described shortly. Fourth, the receiver picks $a^R \in \mathrm{A^R}$ and the sender picks $w \in \mathrm{W}$. Finally, $v^i\colon \mathrm{WxA^R} \to \mathcal{R}$ is the utility function of player $i = S, R$. If W and $\mathrm{A^R}$ are finite sets, a finite set of messages $"\mathrm{M}"$ suffices to communicate. Strategies and beliefs are given by $(\omega^S, \sigma^S, \sigma^R, \mu)$, where:

- A strategy for the sender is: a vector of probability distributions $\omega^S = (\omega^S(w_1), \dots, \omega^S(w_W))$, where the messages sent to announce each move $w \in \mathrm{W}$ are given by $\omega^S(w) = (\omega^S(w)("m_1"), \dots, \omega^S(w)("m_M"))$, a probability distribution on $"\mathrm{M}"$, i.e., $\omega^S(w)("m") \in [0,1]$ and $\sum_{"m" \in "\mathrm{M}"} \omega^S(w)("m") = 1$; and a vector of probabilities $\sigma^S = (\sigma^S(w_1), \dots, \sigma^S(w_W))$, where each move $w \in \mathrm{W}$ has probability $\sigma^S(w) \in [0,1]$ and $\sum_{\mathrm{w} \in \mathrm{W}} \sigma^S(w) = 1$.
- A strategy for the receiver is a vector of probability distributions $\sigma^R = (\sigma^R("m_1"), \dots, \sigma^R("m_M"))$, where the moves chosen in response to each message $"m" \in "\mathrm{M}"$ are given by $\sigma^R("m") = (\sigma^R("m")(a_1^R), \dots, \sigma^R("m")(a_A^R))$, a probability distribution on A^R, i.e., $\sigma^R("m")(a^R) \in [0,1]$ and $\sum_{a^R \in \mathrm{A}^R} \sigma^R("m")(a^R) = 1$.
- A belief for the receiver is a vector of probability distributions $\mu = (\mu("m_1"), \dots, \mu("m_M"))$, where the beliefs in response to each message $"m" \in "\mathrm{M}"$ are given by $\mu("m") = \left(\mu("m")(w_1), \dots, \omega^S("m")(w_W)\right)$, a probability distribution on W, i.e., $\mu("m")(w) \in [0,1]$ and $\sum_{\mathrm{w} \in \mathrm{W}} \mu("m")(w) = 1$.

Meaningful talk is part of a semiotic-inferential process. In the semiotic step, the key issue is comprehensibility. Formally, *meaningful talk* is a bijection from the powerset of $\widehat{\mathbb{W}}$ to a common language $"\mathbb{M}"$, $e\colon \mathrm{P}(\widehat{\mathbb{W}}) \to "\mathbb{M}"$. One direction, $"\mathrm{Q}" = e(\widehat{\mathrm{Q}})$, denotes the encoding stage by which the sender describes in words an intended set of actions $\mathrm{Q} \subset \mathbb{W}$. The other direction, $\widehat{\mathrm{Q}} = e^{-1}(\text{"Q"})$, denotes the decoding stage by which the words $\text{"Q"} \in \text{"}\mathbb{M}\text{"}$ are interpreted by the receiver

ASSUMPTION 1: The receiver can understand (*comprehend*) the literal meaning $\widehat{m}$ of a message "m" uttered by the sender if and only if it belongs to a common natural language "$\mathbb{M}$".

As long as the speaker sticks to the conventions in a common language "$\mathbb{M}$", messages will be comprehensible; this includes fiction, lies, and economic models.[3] *Incomprehensible* messages are messages $\text{"}m\text{"} \not\ni \text{"}\mathbb{M}\text{"}$.

In the inferential step, the key issue is trust.[4] A sender's *truth-function* is a function $T^S\colon \text{"M"xW} \rightarrow \{0,1\}$, where for type $w \in \mathrm{W}$ and $\text{"}m\text{"} = \text{"Q"}$, $T^S(\text{"}Q\text{"}, w) = 1$ if and only if $w \in \mathrm{Q}$; $T^S(\text{"}m\text{"}, \mathrm{w}) = 0$ otherwise. A receiver's *trust-function* is a function $B^R\colon \text{"M"} \rightarrow \{0,1\}$, where $B^R(\text{"}m\text{"}) = 1$ if "m" is trusted and $B^R(\text{"}m\text{"}) = 0$ if not. Whether the receiver trusts the message or not depends on the strategic context.

ASSUMPTION 2: The receiver may either *trust* the literal meaning of a message uttered by the sender — $B^R(\text{"}m\text{"}) = 1$ — and use it to update the priors, or not — $B^R(\text{"}m\text{"}) = 0$.

Only relevant messages add information to the priors. *Relevant* messages refer to a set S such that $\mathrm{S} \cap \mathrm{W} \neq \emptyset$ and $\mathrm{S} \cap \mathrm{W} \neq \mathrm{W}$. For example, if the sender says "I am in New York" in Schelling's game of *rendez-vous* below, this is the same as no news.

B. Equilibrium: Definition and Implications

We single out a Nash equilibrium of the game with imperfect information to characterize the equilibrium moves of the sender, $\mathrm{W} = \mathrm{A}^{\mathrm{S}}$, as the possible worlds the receiver may face, in analogy to the idea of common priors in incomplete information games. There is no point in talking if the default equilibrium is Pareto optimal for the sender. In the examples, we generally take the common priors to correspond to the strategy p^S of the sender in the least informative mixed-strategy Nash equilibria of the game, and the corresponding strategy p^R

[3] As Wittgenstein (1953: 19) puts it, "Excalibur has a sharp blade" makes perfect sense, whether or not King Arthur's sword exists. For a linguist like de Saussure, the meaning of a message is crucial.

[4] Here the point of view of Peirce, Frege, and other logicians comes to the fore: the referent of the message must be taken into account to determine if the statement implied by the message is true or not.

of the receiver in that equilibrium. If there are more specific priors or ad-hoc information shared by all players, such as Schelling's (1960) focal points, the default equilibrium without communication would instead be something more informative.

The reference to the game without communication allows capturing the potential role of natural language in implementing an equilibrium.

DEFINITION 1: In an imperfect information game, a *meaningful-talk equilibrium* satisfies conditions (1) through (4):

(1) (i) For each $w \in W$,

$\tilde{\omega}^S(w) = \arg\max_{\omega^S(w)} \sum_{"m"} \omega^S(w)("m") \sum_{a^R} v^S(w, a^R) \tilde{\sigma}^R("m")(a^R)$.

(1) (ii) $\tilde{\sigma}^S = \arg\max_{\sigma^S} \sum_w \sigma^S(w) \sum_{"m"} \tilde{\omega}^S(w)("m") \sum_{a^R} v^S(w, a^R) \tilde{\sigma}^R("m")(a^R)$.

(2) For each $"m" \in "M"$,

$$\tilde{\sigma}^R("m") = \arg\max_{\sigma^R("m")} \sum_{a^R} \sigma^R("m")(a^R) \sum_w v^R(w, a^R) \tilde{\mu}("m")(w).$$

(3) If for a message $"m" \in "M"$, there exists a $w = w_i \in W$ such that $\tilde{\omega}^S(w_i)("m") > 0$, then either all messages on the equilibrium path are trusted or none is:

(i) If message $"m" = "Q" \in "M"$ is comprehensible, relevant, and $B^R("m") = 1$, then $\tilde{\omega}^S(w)("m") = 1$ for $w \in Q$, $\tilde{\omega}^S(w)("m") = 0$ for $w \notin Q$, and $\tilde{\mu}("m")(w) = \frac{\tilde{\sigma}^S(w)}{\sum_w \tilde{\sigma}^S(w)}$ for $w \in Q$, $\tilde{\mu}("m")(w) = 0$ for $w \notin Q$ (trusted messages are true).

(ii) If message $"m" \in "M"$ is incomprehensible, irrelevant, or $B^R("m") = 0$, then $\tilde{\mu}("m")(w) = \frac{p(w)}{\sum_w p(w)}$ (beliefs are given by priors); furthermore, the receiver's expected utility from trusting the literal meaning of a relevant message $"m"$, $B^R("m") = 1$, and acting on those beliefs must be lower than that from not trusting it, $B^R("m") = 0$, and acting on the priors.

(4) If for a message $"m" \in "M"$, $\tilde{\omega}^S(w)("m") = 0$ for all $w \in W$, then:

(i) For a message $"m" = "Q" \in "M"$ that is comprehensible, relevant, and $B^R("m") = 1$, $\tilde{\mu}("m")(w) > 0$ for $w \in Q$, $\tilde{\mu}("m")(w) = 0$ for $w \notin Q$.

(ii) For a message $"m" \in "M"$ that is incomprehensible, irrelevant, or $B^R("m") = 0$, $\tilde{\mu}("m")(w) \in [0,1]$ and $\sum_w \tilde{\mu}("m")(w) = 1$.

Conditions (3) and (4) of a MTE differ from a PBE, where the following conditions hold instead:

(3') If for a message $"m" \in "\mathrm{M}"$, there exists a $w = w_i \in \mathrm{W}$ such that $\tilde{\omega}^S(w_i)("m") > 0$, then $\tilde{\mu}("m")(w_i) = \frac{\tilde{\omega}^S(w_i)("m")\tilde{\sigma}^S(w)}{\sum_w \tilde{\omega}^S(w)("m")\tilde{\sigma}^S(w)}$.

(4') If for a message $"m" \in "\mathrm{M}"$, $\tilde{\omega}^S(w)("m") = 0$ for all $w \in \mathrm{W}$, then $\tilde{\mu}("m")(w) \in [0,1]$ and $\sum_w \tilde{\mu}("m")(w) = 1$.

The main difference between both equilibrium concepts has to do with conditions (3) and (3'). Whereas the assignment of actions to messages in (3') has nothing to do with the messages themselves because they are devoid of meaning, as in the PBE of any signaling game, in a MTE the assignment in (3) depends on the literal meaning of the message and whether the receiver trusts the message or not, so it is specifically tailored to language games.

The receiver either trusts all, or none, of the credible messages related to a given equilibrium. What matters for informative equilibria are relevant messages.

DEFINITION 2: An *optimistic* equilibrium is a MTE where $B^R("m") = 1$ for some relevant message $"m" \in "\mathrm{M}"$.

DEFINITION 3: A *pessimistic* equilibrium is a MTE where $B^R("m") = 0$ for all messages $"m" \in "\mathrm{M}"$.

Messages are credible if the sender is willing to choose that actual move when these messages are trusted by the receiver:[5]

DEFINITION 4: A message is *credible* if there is a MTE where the message is either on the equilibrium path and true, or off the equilibrium path, when it is interpreted in its literal sense.

It is trivial to establish that MTE exist.

LEMMA 1: *In imperfect information games, MTE always exist.*

PROOF: *This is trivial because uninformative equilibria always exist where the receiver disregards all messages, so the sender has no incentive to choose a message that is conditional on its move, and vice-versa.*

The notion of relevance allows characterizing *informative* equilibria, where the receiver changes beliefs after some message on the equilibrium path (Sobel 2011: 5).

[5] Demichelis and Weibull (2008: 1304) call these credible messages in imperfect information games *self-committing*, because the sender has an incentive to carry out the strategy when the receiver trusts the corresponding message.

LEMMA 2: *In imperfect information games, informative MTE exist if there are optimistic equilibria where the sender has an incentive to utter credible messages that are relevant.*

PROOF: *If the credible messages on the equilibrium path are relevant, the priors will be affected*.

In an informative equilibrium, conformity between what the receiver literally trusts and the underlying equilibrium strategies of the game is required. It is "as if" the required equilibrium constellation is sparked off by verbal communication. A pedestrian way to achieve this, without requiring any imagination on the part of the receiver, would be for the sender to add a reminder about the intended equilibrium that accompanies the message. This resembles the discussion in Myerson (1989), where the sender may promise to do something, or suggest that the receiver do something.

C. Equilibrium Refinement: Definition and Implications

The key idea for the equilibrium refinement is that, since the sender voluntarily picks a message, the message must be interpreted in light of the sender's preferences. In other words, instead of taking the beliefs as exogenously given by one of the potential equilibria, the sender explicitly picks its most preferred equilibrium (or any of them, should there be ties).[6]

DEFINITION 5: In an imperfect information language game, the equilibrium refinement selects the equilibrium that is (weakly) preferred by the sender.

This equilibrium refinement is in the spirit of an early example in Schelling (1960: 59) on how unilateral communication benefits the sender: one player announces his position and states that his transmitter, but not his receiver, works, adding that he will wait where he is, staying put until the other arrives. In the present framework, the Schelling example implies that the sender is selecting, among the myriad of possible meeting places, the one that

[6] Jorge Streb and Fernando Tohmé (2015) propose a related refinement for signaling games, though its bite is more limited because it only works when all the different sender types are of one mind in relation to the preferred PBE in incomplete information games.

implies walking less. A related coordination game, *rendez-vous*, is discussed in detail in the next section.[7]

The refinement has immediate implications for whether informative or uninformative equilibria subsist.

THEOREM 1: *In imperfect information games, the sender selects an informative equilibrium only if it is at least as well off as in an uninformative equilibrium.*

PROOF: *Suppose that the necessary condition is not satisfied, so the sender is strictly worse off in an informative equilibrium. Once the sender does not take beliefs as given, it can pick instead irrelevant messages from an uninformative equilibrium to improve its payoffs.*

THEOREM 2: *In imperfect information games, the sender selects an uninformative equilibrium only if it is at least as well off as in an informative equilibrium.*

PROOF: *Again, this follows directly from the application of the refinement.*

This refinement leads to an outcome somewhat similar to the agenda-setter model in Thomas Romer and Howard Rosenthal (1978), where the agenda setter can propose an alternative subject to the restriction that the proposal must not be worse than the status-quo for the veto player. Given those restrictions, the agenda setter proposes its preferred alternative. However, credibility is not at stake, so this implies that the proposals are binding for the sender, i.e., they are not mere words like here but rather speech acts.

III. Examples

Some classic examples are used to illustrate how MTE and the equilibrium refinement work.

A. Rendez-Vous: The Pervasive Use of Natural Language

In Schelling's (1960: 55-56) famous example of tacit coordination that involves two people who have to meet in an unspecified spot of New York, at an unspecified hour, the number of meeting times and places is unbounded. Instead of tacit coordination, consider

[7] Demichelis and Weibull (2008:1303-4) , in a setup with costly talk, remark that unilateral communication in imperfect information games tends to lead play toward the Nash equilibrium preferred by the sender, a nice insight that is not pursued further.

what happens if it is possible to briefly talk beforehand over the phone. Consider first the version of *rendez-vous* in Table 1 with only two potential moves: left (l) and right (r). Let the meeting time be at twelve noon.

[Insert Table 1 Here]

In the game without communication, there are two pure strategy Nash equilibria, (l, l) and (r, r), as well as a mixed strategy Nash equilibrium where l is played with probability $p(l) = 1/2$ and r with probability $p(r) = 1/2$. If any of the pure strategy equilibria were expected by both players, there would be no point in engaging in explicit communication, so we take the mixed strategy equilibrium to define the priors.

In the game with communication, the sender (say, row) can send a message. MTE parallel Nash equilibria. First, there are uninformative equilibria where the outcome corresponds to the mixed strategy Nash equilibrium: if the receiver disregards all messages, it is a best response for the sender to babble, and vice-versa. There are also two informative pure-strategy equilibria: one where the sender says "l" (“move left”) when it picks l, another where it says "r" (“move right”) when it picks r. There is also an informative equilibrium that has no parallel in the game without communication, in which the sender mixes between announcing "l" and playing l with probability $p(l)$, and announcing "r" and playing r with probability $p(r)$. Since both players can coordinate actions, the payoffs correspond to a correlated equilibrium (Aumann 1974).

In Figure 1, if the sender plays l when it announces "r", the outcome will be either a payoff of 0 for both players, if the receiver trusts the message, or a payoff of $1/2$, if the receiver disregards the message and sticks to the priors. Hence, unlike PBE, and as in Streb and Torrens (2015), an informative equilibrium is not possible if messages are not used in their ordinary sense.

[Insert Figure 1 Here]

In *rendez-vous* it is possible to strip down the setup to its bare essentials, ignoring the specific moves and messages, because the sender and the receiver are indifferent in regards to any of the meeting places, and if they do not meet the payoff is always the same. Figure 2 graphically represents this when there are a (finite) number of locations $N > 1$. The priors of the game without communication are again that any location is equally likely. The representation shows whether the message uttered by the sender is truthful and whether the receiver is trusting (in other asymmetric information games this drastic simplification is not possible). The sender may reveal the whole truth about the meeting place or choose instead a misleading message, while the receiver may literally trust the sender's message or disregard it.[8] Optimistic equilibria are informative: the sender reveals its true intentions, and the receiver trusts the message. Pessimistic equilibria are uninformative: messages are not conditional on move, or they are outright misleading, so receivers disregard them.[9]

[Insert Figure 2 Here]

Unlike PBE, the inferential step introduces a restriction on how beliefs may be updated on the equilibrium path, so it is not possible for the receiver to reinterpret the message as meaning something else than the priors when it is considered misleading. In *rendez-vous* in particular, there is no way for the receiver to interpret the direction of the bias. If the message "Meet me at noon at the information booth in Grand Central Station" is not trusted, instead of second-guessing whether this message might instead mean something else like "Meet me at 9 a.m. in the lobby of the Chrysler Building", the receiver returns to its diffuse priors that any place is equally likely.

More generally, this example can also represent decentralized markets like the market for lemons where seller and buyer have to coordinate a meeting time and place. Despite the common interests, MTE, just like PBE, does not allow ruling out uninformative equilibria. This does not reflect the widespread use of language to coordinate actions. Indeed, language is so ubiquitous that Adam Smith relates human cooperation through markets to

[8] The sender has an incentive not only to be truthful, but also to be precise as possible. Later on we discuss the issue of imprecise messages. The sender, using mixed strategies, can achieve intermediate degrees of truthfulness which range from stating the plain truth to being deceitful; the dividing line between helpful and misleading messages is when the seller says the truth $1/N$th of the time, which implies that messages are uninformative. The receiver, using mixed strategies, can achieve intermediate degrees of trust, which range from taking the sender's message at face value to ignoring it and sticking to the priors; priors are updated whenever the receiver plays strictly mixed strategies.

[9] In regard to misleading messages, see discussion on uninformative equilibrium in Figure 1 when messages are not used in their ordinary sense. More generally, there are uninformative equilibria where the seller says the truth less than $1/N$ of the time, because the buyer cannot reinterpret the messages beyond what is implied by the priors. There is also an equilibrium in mixed strategies where the seller says the truth with probability 1/N, while the buyer disregards the message; it is not an equilibrium for the receiver to trust the message, because then the sender will always want to say the truth.

our “faculties of reason and speech” (*Wealth of Nations*, book I, chapter 2; see also Ángel Alonso-Cortés 2008).

With the equilibrium refinement, thing change drastically in *rendez-vous*: of all the possible equilibria, the sender has an incentive to use linguistic signs to point to the most informative equilibrium of all that indicates the precise location. This leads to a correlated equilibrium à la Aumann where the sender may select any of the locations randomly and verbally inform the receiver of the intended choice.[10]

B. Battle of the Sexes: When Row Takes the Lead

The battle of the sexes has two pure strategy Nash equilibria, and a mixed strategy equilibrium where each player picks its preferred strategy 2/3 of the time, for row going to a shopping mall, for column a football match.

[Insert Table 2 Here]

The mixed-strategy Nash equilibrium can be taken as the fall-back position in pessimistic equilibria where priors are not updated through communication. The two pure-strategy Nash equilibria in the battle of the sexes are the outcome of optimistic equilibria: *shopping* is the outcome if that message is interpreted as being literally true by the receiver, and the response to any out-of-equilibrium message is to disregard it and return to the priors; similarly for *football*.

Verbal communication as such does not add much, simply replicating the outcomes of the game without communication. Once the equilibrium refinement is introduced, the sender selects the unique pure-strategy equilibrium it prefers, which in row’s case is *shopping*.

C. Prudent Alice: Settling on the Best Nash Equilibrium

Consider the imperfect information game devised by Aumann (1990) where prudent Alice prefers to play safe and choose d even if she and Bob verbally agreed to play c.

[Insert Table 3 Here]

[10] In *rendez-vous*, the Demichelis and Weibull (2008) lexicographic misrepresentation costs, a minimalist version of costly talk, also lead to the most informative equilibrium.

Demichelis and Weibull (2008: 1298) show, in a communication game within an evolutionary setup where the two players make their announcements simultaneously, that there is an equilibrium where both players announce "d" and play c when the other player announces "d". Unlike evolutionary games, this convention cannot be established in one-shot games. The problem is quite different: the literal meaning is exogenously given and what is at stake is whether to trust the message or not.

Let the priors of the game without communication be given by the (d, d), instead of the diffuse mixed-strategy Nash equilibrium where c is played with probability 7/8 and d with probability 1/8.[11] These priors capture some of the flavor of Aumann's story, where Alice wants to play d no matter what Bob says. In the current framework, there is a pessimistic equilibrium where Bob announces "c" but both players pick d. The situation where Bob announces "c" and plays c, while Alice plays d, is of course possible, but it is not an equilibrium.

In this setup, the equilibrium refinement leads to select the Pareto-superior optimistic equilibrium where Bob truthfully announces "c" and prudent Alice trusts the message.

D. Precision and Relevance

Rendez-vous is the best-case scenario for successful communication where the sender wants to be as precise as possible. In other strategic situations, imprecise messages may help information transmission take place. Consider the game in Table 4 where there is a pure-strategy Nash equilibrium, (l, l), and two mixed-strategy Nash equilibria where both players pick either mixed strategies (0, 1/2, 1/2) or (9/11, 1/11, 1/11).

[Insert Table 4 Here]

Let the equilibrium without communication be given by the Nash equilibrium with the most diffuse priors, where the beliefs are that the three strategies are played with probabilities (9/11, 1/11, 1/11). The set of precise messages { "l" }, {"m" }, {"r" } only allows to reach the pure strategy equilibrium where l is played. If the partition { "l" }, { "m" or "r"} is considered instead, it is possible to reach the Pareto-superior mixed

[11] If the priors where given by the pure strategy Nash equilibrium (c,c), there would be no point in talking.

strategy equilibrium where m or r are played with probability 1/2. This, of course, follows the thrust of Crawford and Sobel's (1982) pioneering contribution on the most informative partition achievable for different degrees of bias of the sender.

Crawford and Sobel's (1982) idea of strategic information transmission contrasts with Paul Grice's (1975) Cooperative Principle. Grice's (1975:45–47) four maxims on the Cooperative Principle are well known (Ariel Rubinstein 2000 discusses them in chapter 3). The maxims are: informativeness, truthfulness, relevance, and perspicuity. Strategic analysis shows that these maxims do not hold in all games. Rather, they can be interpreted as the optimistic equilibrium of a pure coordination games such as *rendez-vous*. Grice (1975: 45) is aware of this, because he explicitly considers talk exchanges in which there is a common purpose, or at least a mutually accepted direction.

Sperber and Wilson (1995: 268) characterize Grice's Cooperative Principle as a principle of maximal relevance, which will not be satisfied in many situations because the interests of the sender will limit the amount of information it will be willing to reveal. Though they do not have an explicit strategic setup, Sperber and Wilson (1995: 270) propose a principle of optimal relevance instead. This perfectly characterizes the equilibrium refinement proposed in this paper. The sender aims at optimal relevance, not at maximal relevance. Another way of putting this is that the sender will select the optimal degree of precision. Theorem 1 above states a necessary condition for informative equilibria to be selected (if they exist).[12]

IV. Equilibrium Meaning: A Formal Pragmatics

The focus in this paper is not on language as a convention, but rather on how language is employed as a means of communication. Another way of putting this is to distinguish between the semantic and pragmatic planes. From the semantic point of view in linguistics, semiotics, and philosophy of language, sentences have a literal meaning. From the pragmatic point of view, the actual interpretation of linguistic signs depends on each concrete situation. This relates to the distinction Ludwig Wittgenstein (1953) draws

[12] Matching pennies, a zero-sum game, is a case in point: if both players call "heads" or "tails" at the same time, row wins, else column wins. Verbally communicating intentions beforehand is useless.

between a grammar, i.e., norms for meaningful language, and language games, i.e., activities where language is used (Anat Biletzki and Anat Matar 2009).

Wilson and Sperber (2012: 1-10) contrast the ordinary language philosophy in the tradition of the later Wittgenstein, Austin, and Strawson, which analyzes actual language use in all of its complexity, to ideal language philosophy in the tradition of Frege, Russell, Carnap, and Tarski, which treats sentences as encoding something close to full propositions.[13] In giving the actual meaning of a word, Wittgenstein (1953: 21) states: "The meaning of a word is its use in the language." Wittgenstein (1953: 31) considers that any explanatory generalization should be replaced by a description of its use: "don't think, but look!" (Biletzki and Matar 2009). Hence, Wittgenstein (1953) proposes to study language games in all its richness.

In studying language games, this paper leaves aside pragmatic issues like irony and metaphorical uses of speech, and the vagueness, incompleteness and ambiguity of actual speech (Wilson and Sperber 2012). The point it tries to make is different: even if the sender has messages to encode all the relevant information, the literal meaning still cannot be taken at face value because the senders' incentives to be truthful depend on the strategic context of each specific game.

At a highly abstract level, the use of words (i.e., the equilibrium meaning) varies with the strategic incentives in each game: words at times are literally true, at others they must be interpreted in terms of the priors of the game.[14] Given that utterance comprehension is studied within an idealized strategic context, the present approach is a formal pragmatics.[15]

This formal pragmatics extends the traditional semiotic model of communication: while the encoding-decoding step corresponds to the semiotic, or code, model of communication, the inferential step introduces strategic considerations that determine whether the message is trusted by the receiver or not. This can be related to Sperber and Wilson (1995:2), who oppose the traditional code model of communication to the inferential model of Paul Grice

[13] Wilson and Sperber (2012: 1-10) identify an intermediate position, that of Grice, Lewis, and Searle, which distinguishes between sentence meaning and speaker meaning. Sentence meaning, or literal meaning, is still considered to encode something close to a full proposition, with reference assignment being needed to yield a full proposition.

[14] Parikh (2010) discusses the equilibrium meaning of language, but his main concern is about the costs for the sender of being more precise, not about credibility. In this regard, the cost-benefit approach in Sobel (2011: 30-33) offers an interesting approach to describe and interpret information.

[15] Michael Franke (2013) has a survey of this subfield of game-theoretic pragmatics.

and David Lewis, where the hearer must infer the speaker's intention from the verbal information that is uttered. Though Sperber and Wilson (1995) do not use a formal game-theoretic framework, their focus is at times very close to the semiotic-inferential approach presented here.[16]

V. Closing Words

The model of unilateral communication through natural language in Streb and Torrens (2015) is extended here from incomplete information to imperfect information games. The literal meaning of sentences is taken as something given by pre-existing conventions in society, in order to study how linguistic signs may convey information about the sender's intentions to the receiver. Linguistic signs do not provide direct evidence of the sender's intentions, they merely point to them. Hence, a leap of faith is involved in verbal communication: trusting what you can't see in optimistic equilibria. In certain strategic situations, linguistic signs constitute an additional source of information not explicitly formalized before.

An equilibrium refinement is proposed to select among the multiplicity of equilibria of these language games. In the spirit of equilibrium selection in Schelling (1960), the sender specifically uses explicit communication to reach its (weakly) preferred equilibrium. This can be expressed concisely: the sender chooses the optimal degree of precision, in Sobel's (2011) terms; or, the sender aims at optimal relevance, in Sperber and Wilson's (1995) terms. Whether an informative equilibrium is selected or not depends on whether the sender wishes to reveal information or not.

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[16] Sperber and Wilson (1995:175-6) go on to develop an ostensive-inferential model, where the verbal information that the sender manifestly points out by uttering it is used by the receiver to try to infer what the sender means. The ostensive-inferential model exceeds the purely symbolic dimension explored here, since it includes body language, e.g., things like tone of voice to express irony so the exact opposite of what is literally being said reflects the speaker's intention or "propositional attitude" (Sperber and Wilson 1995: 9-11).

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TABLE 1— *RENDEZ-VOUS*

	left (*l*)	*right* (*r*)
left (*l*)	1,1	0,0
right (*r*)	0,0	1,1

TABLE 2— BATTLE OF THE SEXES

	shopping (*s*)	*football* (*f*)
shopping (*s*)	2,1	0,0
football (*f*)	0,0	1,2

TABLE 3— BOB AND PRUDENT ALICE

	c	*d*
c	9,9	0,8
d	8,0	7,7

TABLE 4— GAME WHERE IMPRECISION IS INFORMATIVE

	l	*m*	*r*
l	1,1	0,0	0,0
m	0,0	9,0	0,9
r	0,0	0,9	9,0

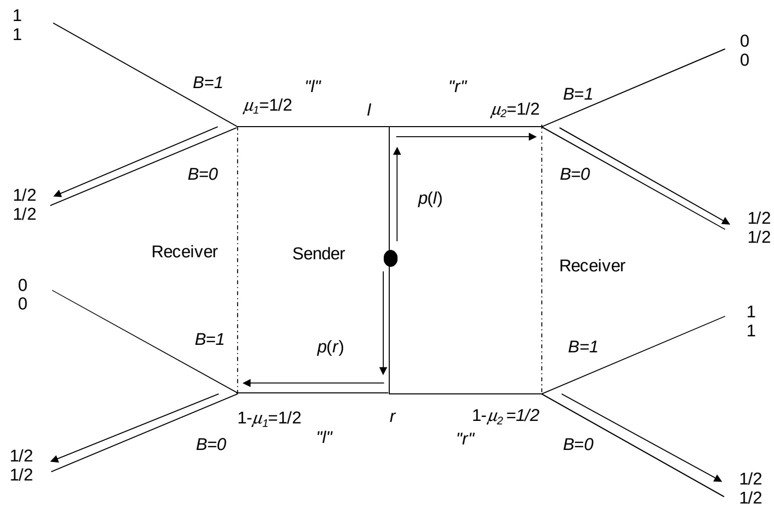

Figure 1. *Rendez-vous*: Uninformative Equilibrium where Message are Disregarded

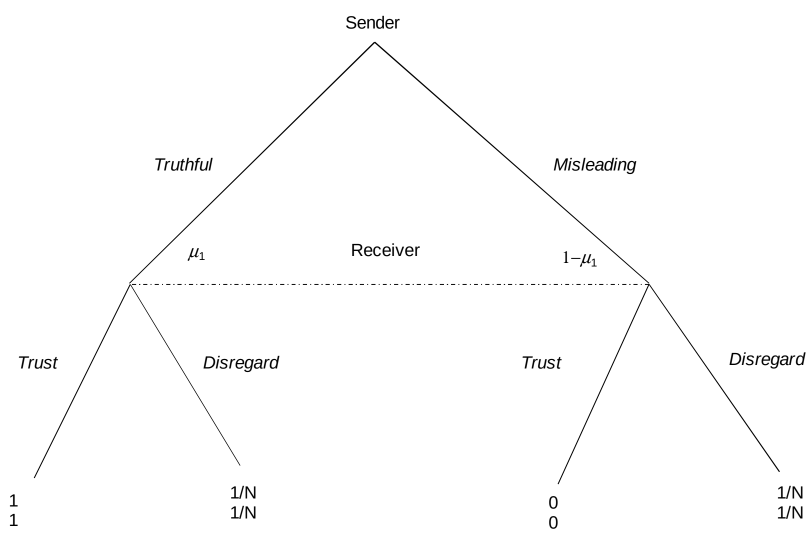

Figure 2. *Rendez-vous*: *N* Possible Meeting Places