UNIVERSIDAD DEL CEMA
Buenos Aires
Argentina

Serie
DOCUMENTOS DE TRABAJO

Área: Economía

HONESTY, LEMONS, AND SYMBOLIC SIGNALS

Jorge M. Streb y Gustavo Torrens

Julio 2012
Nro. 492

ISBN 978-987-1062-79-9
Queda hecho el depósito que marca la Ley 11.723

www.cema.edu.ar/publicaciones/doc_trabajo.html
UCEMA: Av. Córdoba 374, C1054AAP Buenos Aires, Argentina
ISSN 1668-4575 (impreso), ISSN 1668-4583 (en línea)
Editor: Jorge M. Streb; asistente editorial: Valeria Dowding jae@cema.edu.ar

Streb, Jorge Miguel
Honesty, lemons, and symbolic signals. - 1a. ed. - Buenos Aires : Universidad del CEMA, 2012.
38 p. ; 22x15 cm.

ISBN 978-987-1062-79-9

1. Economía. I. Título.
CDD 330

Fecha de catalogación: 19/07/2012

Honesty, lemons, and symbolic signals*

Jorge M. Streb
Universidad del Cema

Gustavo Torrens
Washington University in St. Louis

July 2012

Abstract

Under asymmetric information, dishonest sellers lead to market unraveling in the lemons model. An additional cost of dishonesty is that language becomes cheap talk. We develop instead a model where people derive utility from actions (what they say), as well as from outcomes, so talk is costly. We find that the existence of honest agents that mean what they say is not enough to make trade more likely, unless a traceability condition that prevents arbitrage is met. When we introduce a continuum of misrepresentation cost types and qualities, full market unraveling is not possible and babbling equilibria are eliminated. More generally, costly talk is a special kind of signal, a symbolic signal that presupposes linguistic conventions, otherwise truth and falsehood, as well as misrepresentation costs, are undefined.

Resumen. Bajo información asimétrica, vendedores deshonestos llevan a la desaparición del mercado en el modelo de los "lemons". Un costo adicional de la deshonestidad es que el lenguaje se vuelve vacío. Desarrollamos en cambio un modelo donde las personas derivan utilidad tanto de sus acciones (lo que dicen) como de las consecuencias, lo que transforma al lenguaje en una señal costosa. La existencia de agentes honestos que dicen lo que piensan no facilita de por si el intercambio, a menos que se cumpla una condición de trazabilidad que evite el arbitraje. Cuando introducimos un continuo de tipos de costos de mentir y de calidades, encontramos que no es posibible la desaparición completa del mercado y se eliminan los equilibrios no informativos. Más en general, el lenguaje costoso es un tipo especial de señal, una señal simbólica que presupone convenciones linguísticas, sino la verdad y falsedad, así como los costos de mentir, no están definidas.

JEL classification codes: D8, C7

Key words: asymmetric information, honesty, trust, symbols, signals, costly talk

1 Introduction

The standard view in economics is that misrepresentation is costless. In Joseph Schumpeter's (1942: 264) paradoxical terms, "Since the first thing man will do for his ideal or interest is to lie, we shall expect, and as a matter of fact we find, effective information is almost always adulterated or selective."[1]

*We appreciate the suggestions by Ignacio Armando, Germán Coloma, Mariana Conte Grand, Alejandro Corbacho, Enrique Kawamura, Augusto Nieto Barthaburu, Francisco Sánchez, Matteo Triozzi, and participants in presentations at Universidad del Cema, and the meeting of JOLATE in San Luis, AAEP in Mendoza, and LACEA in Buenos Aires. We benefitted from conversations with George Akerlof and Matt Rabin.

[1]William Mitchell (1984: 82) discusses how Schumpeter applies this insight to political advertising.

Vincent Crawford and Joel Sobel (1982) formalize language as cheap talk to study the maximal amount of information an expert (the informed party) may offer a decision maker (the uninformed party) when there are incentives to lie. Cheap talk sets language apart from signals: while signals are credible because choices are differentially costly, language is not because it has no direct payoff consequences (see, e.g., Robert Gibbons 1992).

However, from a psychological viewpoint, words are a highly subjective phenomenon. Thomas Schelling (1960: 26-27) already recognizes that "if part of the population belong to the cult in which 'cross my heart' is (or is believed to be) absolutely binding ... they can commit themselves, the others cannot"; for this cult, words are equivalent to moves, not to speech. But we do not need to rely on a cult of fanatics, since for most people lying is not a free lunch, it has a psychic cost. As Brad Blanton (1996, chapter 5) puts it, lying is stressful and can make us sick. In this same line, Smith's (1759) view on veracity and deceit is that "common lying" is shameful (Part VII, Section IV). Even if we let aside remorse for not acting correctly, being truthful is still less costly than making something up. Indeed, some lie detector tests are based on the idea that lies can be detected by more cerebral activity – this second explanation, however, might not be entirely independent from the first if parts of the brain more associated with negative emotions are also activated (see Stefano Demichelis and Jörgen Weibull 2008, 1293-4).

Regardless of the reasons that justify the existence of misrepresentation costs, when words affect utility they become signals. The idea that some people are willing and able to manipulate language at will, while others stick to their word, has been formalized by Steven Callander and Simon Wilkie (2007), and Navin Kartik, Marco Ottaviani, and Francesco Squintani (2007). Kartik, Ottaviani, and Squintani (2007) point out that once the sender suffers disutility from misreporting private information, the game is transformed from cheap talk to costly signaling. In Callander and Wilkie (2007) there are either honest or dishonest political candidates, and the utility of honest candidates depends on campaign promises. With a continuum of policy preferences, in equilibrium all campaign platforms have some informative content about post-electoral intentions, a result that implies babbling equilibria are eliminated.

In George Akerlof's (1970) market for lemons, all sellers are dishonest, and hence perfectly willing to misrepresent quality.[2] Thus, the model can be rephrased as a cheap talk game. In contrast, we explore what happens to the lemons model when some sellers have a distaste for misrepresenting the truth. In other words, not only the market outcome but also the process followed to reach it affect the utility of some individuals. Hence, besides buyers that cannot distinguish between high and low quality products, we introduce costs of misrepresentation that resemble the honesty costs in Demichelis and Weibull (2008), though honesty there only enters lexicographically to break ties in material gains. We first analyze two types of misrepresentation costs, before introducing a continuum of types. We do this in a setting with two, and then a continuum of types of quality. This setting leads to two-dimensional asymmetric information, because not only quality but also misrepresentation costs are unobservable.

The existence of misrepresentation costs imply there may be trustworthy sellers. For Ronald Coase (1976: 545), "the observance of moral codes must very greatly reduce the costs of doing business with others and must therefore facilitate market transactions", a view which he traces back to Adam Smith. For Douglass North (1981: chapters 4 and 5), honesty, integrity, and gentlemen's agreements help markets

[2]The relevance of differences in character is only hinted at for the specific case of credit markets in underdeveloped countries, where local moneylenders have a personal knowledge of the character of the borrowers that outsiders do not (Akerlof 1970: 498-9).

work, and a system that is viewed as fair and legitimate by its citizens diminishes the costs of enforcement. Cooperation based on trust in what other people say differs from informal mechanisms that explain cooperation through repeated interactions, which is sustained by reputational channels (see Michihiro Kandori 1992; this mechanism is already suggested for markets by Akerlof 1970: 499-500 via brand-names and business chains). It also differs from guarantees (Akerlof 1970: 499-500) because these rely on formal mechanisms, namely a well-functioning legal system that makes guarantees binding for the seller.

As to the market implications of misrepresentation costs, our results can be interpreted in a negative vein. First, in our model the problem of market breakdown under asymmetric information cannot be overcome even with a large proportion of honest sellers, unless an additional condition is satisfied that allows to avoid arbitrage between markets (a traceability condition). Hence, though trust might work in personalized settings, there are severe limits to the relevance of trust on its own for the functioning of impersonal markets. Second, even when arbitrage can be somehow avoided, an increase in misrepresentation costs for some agents can have a weak, or even null, effect on market outcomes, particularly when these misrepresentation costs remain low in comparison to market opportunities.

As to the implications for language, in line with the results on political campaigns in Callander and Wilkie (2007), we find that, even when the proportion of honest sellers is small and does not affect market outcomes, misrepresentation costs imply that language is no longer cheap but rather a costly signal. However, the result in Callander and Wilkie (2007) that babbling equilibria are eliminated with misrepresentation costs only holds with a continuum of quality types. With two quality types, babbling equilibria subsist as long as the largest misrepresentation cost is smaller than the gain from misrepresenting quality – a result that can also be expected to hold with any discrete number of quality types.

Finally, we call the attention to an implicit assumption in our model, and all costly talk models in general. The key idea is that if there is no conventional way of saying things, there can be no cost from distorting messages. In other words, costly talk presupposes a common language whose literal meaning is shared by all the players. Unless a set of symbolic signs are shared by a given community, there is no way to verbally misrepresent information, because there would be no way in the first place for the speaker to verbally represent information and communicate meaning, whether true or false, and of course no way for the listener to understand the messages. Honesty and dishonesty are undefined without this prior conventional meaning that allows to evaluate the truth, or falsehood, of the statements uttered. This implies that costly talk is a special kind of signal, namely, a "symbolic" signal that relies on a shared culture.

The rest of the paper is organized as follows. In Section 2 we describe our framework. In the following three sections we restrict our attention to equilibria with pure price strategies. In Section 3, we show how language is uninformative in the original lemons model, first with two and then with a continuum of quality types. Section 4 introduces two types of misrepresentation costs: some sellers have very high misrepresentation costs, so they are honest, while the rest are dishonest. Section 5 introduces a continuum of misrepresentation cost types. Section 6 shows that mixed price strategy equilibria do not exist, with one exception, the setup with two qualities and a continuum of misrepresentation costs. In this setup, mixed price strategy equilibria exist in particular for parameter values in which no pure price strategy equilibrium exists. Section 7 uses ideas on signs from semiotics to make explicit an assumption implicit in costly talk models. The last section concludes.

2 Cheap and costly talk

Consider a simple version of Akerlof's (1970) market for lemons, augmented with misrepresentation costs. Each seller has a product of quality $\theta \in \Theta$ and a reservation utility of $\alpha\theta$, where $0 < \alpha < 1$. Sellers also differ in their personal character. Each seller has an unobservable characteristic, misrepresentation cost $h \in H$. This cost represents differing willingness, and ability, to cheat others. The utility function of each type of seller depends on the price p of the product offered by the buyer and the message "m" that it sends:

$$U_S\left(\theta, h, \text{``}m\text{''}, p\right) = p - \alpha\theta - C(\theta, h, \text{``}m\text{''}), \tag{1}$$

where the function $C(.)$ represents the personal cost of misrepresentation:

$$C(\theta, h, \text{``}m\text{''}) = \begin{cases} h, & \text{if } \text{``}m\text{''} \neq \text{``}\theta\text{''}, \\ 0, & \text{if } \text{``}m\text{''} = \text{``}\theta\text{''}. \end{cases} \tag{2}$$

Note that we follow the use/mention distinction to distinguish between (i) using a word to refer to the world, e.g., quality type θ, and (ii) mentioning a word as a linguistic symbol, e.g., quality message "θ". That is, we distinguish between two planes, that of reality with types and moves as such (m), and that of language with messages about types and moves ("m").

This specification of the cost function resembles the fixed and heterogenous honesty costs in Demichelis and Weibull (2008), though there honesty only matters in case of ties in material payoffs. Indeed, Demichelis and Weibull (2008) have something slightly weaker than costly talk, but stronger than cheap talk, since their lexicographic ordering implies that only if the material payoffs are equal, then truthfulness is preferred; otherwise, material payoffs guide decisions. The specification is somewhat similar to the misrepresentation costs in Callander and Wilkie (2007) and Kartik, Ottaviani, and Squintani (2007). However, there are two important differences. First, we assume that the process itself of lying or not affects utility, regardless of the outcome, i.e., the cost is incurred even when lying is not effective at fooling the other player. Second, we disregard the size of the lie, i.e., the difference between the announced and the actual quality the sender owns.

The timing is that, after nature determines the quality type of the product and the misrepresentation cost, the seller, who observes both, sends a message about the quality type, to which the buyer answers with a price offer. The seller can then take or leave this offer. Moreover, in order to abstract from the bargaining problem, we assume buyers are risk neutral, and hence willing to pay the average quality offered on the market.[3] We concentrate on misrepresenting quality. In the other steps there is no conflict of interest, only a coordination problem, so we could assume that messages about prices and acceptance are truthful.[4] To simplify matters, we suppose instead that both the price offer by the buyer and the acceptance by the seller are transformed into moves because they are firm offers, namely written offers that legally bind the sender to comply with those terms if the receiver so chooses. We can call these

[3]This follows if demand is perfectly elastic at a price that equals the average quality that sellers effectively offer. It is also possible to derive this in a model where each seller faces two buyers who choose prices à la Bertrand (Andreu Mas-Colell et al. 1995: 442).

[4]Streb and Torrens (2011) analyze coordination games, showing that there is an informative equilibrium where people use natural language in its ordinary sense. However, uninformative equilibria are also possible if expectations are utterly pessimistic.

moves "legal talk", which requires a common natural language shared by all market participants, as well as the observance of the rule of law. Legal talk is thus a signal, but for different reasons than costly talk (for formal, rather than informal, reasons).

Depending on the assumptions about the sets Θ and H we have the following six cases to consider:

Misrepresentation costs\Quality	Two types: $\Theta = \{\theta_L, \theta_H\}$	Continuum: $\Theta = [\theta_L, \theta_H]$
Cheap Talk: $H = \varnothing$	Proposition 1 (Section 3.1)	Proposition 2 (Section 3.2)
Costly Talk: two types: $H = \{0, \chi\}$	Proposition 3 (Section 4.1)	Proposition 4 (Section 4.2)
Costly Talk: a continuum: $H = [0, \chi]$	Proposition 5 (Section 5.1)	Proposition 6 (Section 5.2)

Table 1. Alternative asymmetric information setups

Following the standard approach in the literature on the market for lemons, these propositions are derived using pure price strategies. However, since pure price strategy equilibria may not exist with a continuum of misrepresentation cost types and two quality types, we then look at the extension to mixed price strategies.

3 The lemons model as a cheap-talk game

In this section we rephrase Akerlof's (1970) lemons model as a cheap-talk game where the seller is the sender and the buyer is the receiver. Suppose that all sellers are dishonest ($H = \varnothing$). Hence, messages have no cost for the senders, and the lemons model becomes a cheap-talk game.

3.1 Two quality types

Let's begin with a model with two quality types. Suppose that each seller owns a unit of a product of quality $\theta_i \in \Theta = \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L$. The opportunity cost of each seller is $\alpha\theta_i$ and $\alpha\theta_H > \theta_L$. The quality is known to the seller, but not to the buyer, at purchase time. Buyers know that the product is high quality with probability $q > 0$ and low quality with probability $1 - q > 0$.

Though the seller can potentially send any message, the minimal messages required are the possible qualities of the good. Let "m" $\in \{$"θ_L", "θ_H"$\}$, where "θ_H" can be interpreted as "This is a high quality product", and "θ_L" can be interpreted as "I must warn you this is a lemon". The strategy of the sellers can be written as a function $m_S(\theta_i)$. Let $m_s(\theta_i)$ ("m") denote the probability that message "m" is sent by type θ_i.

For each message "m", the buyer forms a conjecture μ ("m") (θ_i) about the product's quality, which we interpret as the buyer's belief that the seller that sends the message "m" has a product of quality θ_i.

Buyers can pay any price p in the interval $[\theta_L, \theta_H]$. We assume a firm offer, i.e., that the price offer "p" obliges the buyer to pay price p since it is backed by a written proposal that may be executed at the will of the seller. We denote by p_B ("m") the price that the buyer offers in response to quality message "m". Moreover, we assume that buyers are willing to pay the average quality offered on the market. In what follows, we concentrate on pure price strategies.

A seller could verbally communicate with the messages "I accept the price offer" ("a") or "I reject the price offer" ("$\sim a$"), but we directly consider the action to accept (a), which is embodied in a legally binding form that transfers property from the seller to the buyer, else the action is not to accept ($\sim a$). In particular, sellers will be willing to accept a price equal to the average expected quality ($\mathbf{E}_q[\theta] = (1-q)\,\theta_L + q\theta_H$) if and only if :

$$\mathbf{E}_q[\theta] \geq \alpha\theta_H \Longleftrightarrow \alpha \leq \bar{\alpha}_1 = \frac{(1-q)\,\theta_L + q\theta_H}{\theta_H} \Longleftrightarrow q \geq \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}.$$

Since sellers will only accept a price offer greater or equal than their opportunity cost, buyers who are forward looking introduce this restriction in their conjectures. We denote by $\tilde{\mu}(\text{"}m\text{"}, p)(\theta_i)$ the buyer's conjecture that the quality of the product is θ_i when it observes message "m" and offers price p.[5]

One can rule out separating equilibria, because sellers of lemons always have an incentive to mimic sellers of high-quality products. A pooling equilibrium exists in which all sellers pool stating they have a high quality product, "m" = "θ_H", regardless of actual quality. For this message to be a perfect Bayesian Nash equilibrium, buyers must expect that anybody who deviates to "θ_L" has a low quality car; there are other pooling equilibria where the same outcome is supported by out-of-equilibrium beliefs that the conditional probability of high quality products is low enough for condition $\mathbf{E}_q[\theta] \geq \alpha\theta_H$ not to be satisfied. There are also pooling equilibria in "θ_L" that follow a similar logic. In the Appendix we formally compute all the equilibria. The following proposition summarizes the results.

Proposition 1 *Consider the lemons model with two quality types, θ_L and θ_H ($\alpha\theta_H > \theta_L$), dishonest sellers, risk-neutral buyers willing to pay the expected quality of the product, $\bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}$ and $\bar{\alpha}_1 = \frac{(1-q)\theta_L + q\theta_H}{\theta_H}$. Let the possible messages in this cheap-talk game be given by "m" $\in$ {"θ_L", "θ_H"}. Then the perfect Bayesian equilibria within the set of pure price strategies are as follows:*

1. *If $\alpha \leq \bar{\alpha}_1$ (equivalently $q \geq \bar{q}_1$), all the equilibria are Pareto efficient and uninformative. In all these equilibria both qualities are traded at price $\mathbf{E}_q[\theta]$. (i) There are pooling equilibria on either "θ_H" or "θ_L". (ii) There are also hybrid equilibria where the sellers mix between both messages with equal probability.*

2. *If $\alpha > \bar{\alpha}_1$ (equivalently $q < \bar{q}_1$), all the equilibria are Pareto inefficient and uninformative. In all these equilibria only products of quality θ_L are traded at the price θ_L. (i) There are pooling equilibria on either "θ_H" or "θ_L". (ii) There are also hybrid equilibria where the sellers mix between both messages with probabilities that, for either message, lead to a conditional probability of high quality goods lower than $\bar{q}$.*

Proof: See the Appendix. ■

[5] While $\mu(\text{"}m\text{"})$ are the buyer's beliefs about product quality for a seller that announces "m", $\tilde{\mu}(\text{"}m\text{"}, p)$ are the buyer's beliefs about product quality effectively supplied by that seller at price offer p.

3.2 A continuum of quality types

We now turn to a setup with a continuum of quality types $\theta \in \Theta = [\theta_L, \theta_H]$. With a uniform distribution, average quality is $\mathbf{E}[\theta] = \frac{\theta_L+\theta_H}{2}$. We consider as possible messages any "m" $\in$ ["θ_L", "θ_H"].

By Akerlof's (1970) unraveling argument, if all sellers are willing to misrepresent quality, for $\theta_L = 0$ there is complete market breakdown when $\mathbf{E}[\theta] = \frac{\theta_H}{2} < \alpha\theta_H$, i.e., $\alpha > \frac{1}{2}$: the highest qualities drop out of the market, so average quality drops, and the process continues until all qualities vanish from the market. If the lowest quality is $\theta_L > 0$, the conclusion must be slightly amended: only qualities $\left[\theta_L, \frac{\theta_L}{2\alpha-1}\right]$ remain on the market, with average quality (and price) given by $\alpha\frac{\theta_L}{2\alpha-1}$. In the Appendix we formally compute all the equilibria. The following proposition summarizes the results.

Proposition 2 *Consider the lemons model with a continuum of quality types θ uniformly distributed over the interval $[\theta_L, \theta_H]$, dishonest sellers, risk-neutral buyers willing to pay the expected quality of the product and $\bar{\alpha}_2 = \frac{\theta_H+\theta_L}{2\theta_H}$. Let the possible messages in this cheap-talk game be given by "m" $\in$ ["θ_L", "θ_H"] Then, the perfect Bayesian equilibria within the set of pure price strategies are as follows:*

1. *If $\alpha \leq \bar{\alpha}_2$, all the equilibria are Pareto efficient and uninformative. In all these equilibria all qualities in the interval $[\theta_L, \theta_H]$ are traded at the price $p^* = \mathbf{E}[\theta]$. There are pooling equilibria on any message "m" $\in$ ["θ_L", "θ_H"], as well as hybrid equilibria where the sellers pick the messages with a probability that is independent of type.*

2. *If $\alpha > \bar{\alpha}_2$, all the equilibria are Pareto inefficient and uninformative. In all these equilibria only qualities in the interval $\left[\theta_L, \frac{\theta_L}{2\alpha-1}\right]$ are traded at the price $p^* = \frac{\alpha\theta_L}{2\alpha-1}$, unless $\theta_L = 0$ so there is full, instead of partial, market unraveling. There are pooling equilibria on any message "m" $\in$ ["θ_L", "θ_H"], as well as hybrid equilibria where the sellers pick the messages with a probability that is independent of type.*

Proof: See the Appendix. ■

3.3 Alternative messages and beliefs of receivers

In Proposition 1 we are implicitly considering only messages that are comprehensible, i.e., in a language common to sender and receiver, in our case English. Furthermore, we are also requiring that messages be relevant, i.e., that they refer to a quality actually on the market, so the only candidate messages left are "m" $\in$ {"θ_L", "θ_H"}. However, cheap-talk games do not restrict the specific messages used, only the beliefs induced in equilibrium (see, e.g., Wang 2009). Hence, the messages could also be incomprehensible or irrelevant. Anything goes. In other words, when we do not restrict the messages to "m" $\in$ {"θ_L", "θ_H"}, there are countless more equilibrium messages, since any noise that can potentially be uttered by the sender may be an equilibrium message in these babbling equilibria. The reaction to out-of-equilibrium announcements (should there be any) would have to be a low price. Similar remarks apply to Proposition 2.

Can we be a bit more specific in relation to the equilibrium messages in Proposition 1 if we add the restriction that the receiver is left with the option of either believing the message literally or sticking to its priors (Streb and Torrens 2011)? Not always. For $\alpha \leq \bar{\alpha}_1$ (equivalently $q \geq \bar{q}_1$), the message "θ_L" is credible, since that quality is available on the market. Hence, the receiver can either take it literally and offer to pay θ_L, or disregard it and pay more (namely $\mathbf{E}_q[\theta]$). Hence, when "θ_L" is believed, there is an equilibrium where senders have no incentive to send that message; when all messages are disregarded, another equilibrium is possible where either message may be used. For $\alpha > \bar{\alpha}_1$ (equivalently $q < \bar{q}_1$), on the other hand, buyers are willing to pay θ_L whatever the message, so in equilibrium any message is always possible.

Can we be a bit more specific in relation to the equilibrium messages in Proposition 2 if we add the restriction that the receiver is left with the option of either believing the message literally or sticking to its priors? Again, not always. Messages about qualities below the average quality $\frac{\alpha\theta_L}{2\alpha-1}$ on the market are credible, so senders have no incentive to send those messages, unless they are sure the messages will not be believed (if believed, "m" < "$\frac{\alpha\theta_L}{2\alpha-1}$" leads to a price offer lower than the average price $\frac{\alpha\theta_L}{2\alpha-1}$ that buyers are willing to pay). In the case of disbelief, there remain equilibria where any message goes. For $\theta_L = 0$, on the other hand, no restriction on equilibrium messages is possible.

4 Two misrepresentation cost types

In this section we first study two misrepresentation cost types ($H = \{0, \chi\}$), and two quality types ($\Theta = \{\theta_L, \theta_H\}$). We assume the cost of lying χ is larger that the greatest potential benefit $\theta_H - \theta_L$, so some agents are honest by construction; otherwise, the results of Proposition 1 would not be affected.[6] We then apply this assumption to a setting with a continuum of quality types ($\Theta = [\theta_L, \theta_H]$). Since we are treating with impersonal markets, a traceability condition is required to prevent arbitrage between markets.

4.1 Two quality types

Suppose that there are two quality types $\theta_i \in \Theta = \{\theta_L, \theta_H\}$, and two quality types $h \in H = \{0, \chi\}$. We assume that quality and honesty are independently distributed, though the only crucial requirement is that the probability of honest sellers with lemons be positive: there is a fraction $q \in (0,1)$ of high-quality products, and $(1-r) \in (0,1)$ of honest sellers with $\chi \geq \theta_H - \theta_L$ who derive utility from the outcomes and also the actions they take.[7] The lemons model with honest sellers is no longer a cheap-talk game, but rather a signaling game. Now, the strategy of the sellers is a function of its type $t = (\theta_i, h)$, i.e., $m_S(t) \in \{$"θ_L", "θ_H"$\}$. Buyers still make a price offer p_B("m"), which sellers can accept or not.

The presence of honest sellers can help to tie the equilibrium meaning of messages to their literal meaning. In the original market for lemons model, honest sellers who have a high quality product find their announcements "θ_H" are not credible, because dishonest sellers with a low quality product claim the same thing. This credibility problem does not arise with honest sellers who have a low quality product, because the announcement "θ_L" is perfectly credible. This creates two different markets. The first market, associated to the message "θ_L", is a standard competitive market where the quality of the commodities transacted is certain. The equilibrium boils down to a competitive equilibrium where $(1-q)(1-r)$ low quality units are transacted at a price θ_L.

[6] The condition for sellers to voluntarily reveal the quality of their product can be somewhat relaxed.

[7] Stein and Streb (2004) have a similar setup with two-dimensional asymmetric information, where governments differ in competence and opportunism.

In the second market, associated to the message "θ_H", purchasers need to deduce the expected quality of what they are buying using the information they have about the sellers' behavior. The presence of honest sellers raises average quality in the second market. Instead of $\mathbf{E}_q[\theta] = (1-q)\theta_L + q\theta_H$, average quality is now $\mathbf{E}_{q,r}[\theta] = \frac{(1-q)r\theta_L + q\theta_H}{(1-q)r+q}$. A seller can accept ($a$) or not ($\sim a$) the price offer made by the buyer. In particular, sellers will be willing to accept a price equal to the average expected quality if and only if

$$\mathbf{E}_{q,r}[\theta] \geq \alpha\theta_H \iff \alpha \leq \bar{\alpha}_3 = \frac{(1-q)r\theta_L + q\theta_H}{(1-q)r\theta_H + q\theta_H} \iff q \geq \bar{q}_3 = \frac{r(\alpha\theta_H - \theta_L)}{(1-\alpha)\theta_H + r(\alpha\theta_H - \theta_L)}.$$

Thus, when $\alpha \leq \bar{\alpha}_3$ (equivalently, $q \geq \bar{q}_3$) there is a unique semi-separating equilibrium: "θ_L" conveys the information that the product is a lemon, while "θ_H" is associated to a sufficiently high proportion of high quality products.

If $\alpha > \bar{\alpha}_3$ (equivalently, $q < \bar{q}_3$), there is market breakdown and $p_B(\text{“}\theta_H\text{”}) = \theta_L$ in this second market as well. A semi-separating equilibrium exists where honest sellers that have lemons pick "θ_L", while the other sellers pick "θ_H". Because dishonest sellers are indifferent in equilibrium between both markets, this same outcome is supported by a continuum of equilibria where some dishonest sellers of lemons say they have a lemon, as long as the proportion of dishonest sellers that are truthful is low enough for $\alpha \leq \bar{\alpha}_3$ not to hold in market "θ_H".

A pitfall of the semi-separating equilibrium when $\alpha \leq \bar{\alpha}_3$ is that dishonesty pays: a dishonest buyer can buy a lemon in market 1 and resell it in market 2, making a profit of $\mathbf{E}_{q,r}[\theta] - \theta_L$. Hence, dishonest sellers have an incentive to swamp market 1, reselling the good in market 2, thus undoing the presence of honest sellers of lemons that separate out in market 1. To avoid this, we need a traceability condition:

Definition 1 *Traceability condition: buyers can recognize which goods have been sold as lemons.*

In our market example, buyers can resort to a specific sign to avoid arbitrage: they can check whether the seller is the original owner, instead of a middleman that just bought the good in market 1 ("θ_L") and is reselling it on the spot in market 2 ("θ_H") to make arbitrage profits. The ownership record works as a sign (more specifically, an indexical sign, as discussed in Section 7) that provides evidence on the quality of good, since owners offer lemons with probability $\frac{(1-q)r}{(1-q)r+q} < 1$, while middlemen do so with probability 1. Buyers can avoid arbitrage only if they can condition their offer on this information.

In the Appendix we formally compute all the equilibria. The following proposition summarizes the results.

Proposition 3 *Consider the lemons model with two quality types, θ_L and θ_H ($\alpha\theta_H > \theta_L$), some honest sellers ($(1-r) \in (0,1)$) for whom the cost of misrepresentation is $\chi \geq \theta_H - \theta_L$, risk-neutral buyers willing to pay the expected quality of the product, $\bar{q}_3 = \frac{r(\alpha\theta_H - \theta_L)}{(1-\alpha)\theta_H + r(\alpha\theta_H - \theta_L)}$ and $\bar{\alpha}_3 = \frac{(1-q)r\theta_L + q\theta_H}{(1-q)r\theta_H + q\theta_H}$.*[8] *Let the possible messages in this costly-talk game be given by "m" $\in \{$"θ_L", "θ_H"$\}$. Then the perfect Bayesian equilibria within the set of pure price strategies are as follows:*

[8]The requirement $\chi \geq \theta_H - \theta_L$ can be relaxed to $\chi \geq \mathbf{E}_{q,r}[\theta] - \theta_L = \frac{(1-q)r\theta_L + q\theta_H}{(1-q)r+q} - \theta_L$ to derive the results in Proposition 3. Otherwise, we revert to the results in Proposition 1.

1. *If $\alpha \leq \bar{\alpha}_3$ (equivalently $q \geq \bar{q}_3$), there is a unique Pareto efficient and partially informative equilibrium if the traceability condition is satisfied. In this equilibrium there are two different markets: in the first market, where the message is "θ_L", only products of quality θ_L are traded at price θ_L; in the second market, where the message is "θ_H", both qualities are traded at price $\mathbf{E}_{q,r}[\theta]$. Hybrid equilibria are not possible, since dishonest sellers always want to state "θ_H".*

2. *If $\alpha > \bar{\alpha}_3$ (equivalently $q < \bar{q}_3$), then all equilibria are Pareto inefficient and uninformative. In these equilibria only products of quality θ_L are traded at price θ_L. The messages may be in pure strategies, but there are also hybrid equilibria where dishonest sellers mix between "θ_H" and "θ_L".*

Proof: See the Appendix. ■

Comparing Proposition 1 with Proposition 3, it is easy to see that misrepresentation allows us to extend the region for which it is possible to support a Pareto efficient allocation. In particular, note that $\bar{\alpha}_1 < \bar{\alpha}_3$ (equivalently $\bar{q}_1 > \bar{q}_3$). The Pareto efficient equilibria in the range $(\bar{\alpha}_1, \bar{\alpha}_3]$ can be described as trust equilibria, because they are made possible by the existence of honest agents. When a Pareto efficient allocation can be supported, misrepresentation costs also eliminate the multiplicity of equilibrium messages. However, when there is market breakdown, honesty does not prevent a multiplicity of equilibrium messages, because the behavior of dishonest sellers is not completely pinned down.

4.2 A continuum of quality types

Suppose that there is a continuum of quality types $\theta \in \Theta = [\theta_L, \theta_H]$, and two honesty types $h \in H = \{0, \chi\}$, with $\chi \geq \theta_H - \theta_L$. Quality is uniformly distributed with density $(\theta_H - \theta_L)^{-1}$, while the probability of an honest type is $(1-r) \in (0,1)$.

We look for an equilibrium with a price strategy of the following form:[9]

$$p_B(\text{"}m\text{"}) = \begin{cases} m & \text{if "}m\text{"} \in [\text{"}\theta_L\text{"}, \text{"}\theta^*\text{"}], \\ p^* & \text{if "}m\text{"} \in (\text{"}\theta^*\text{"}, \text{"}\theta_H\text{"}]. \end{cases}$$

That is, in order to avoid arbitrage, all announcements above a certain threshold "θ^*" receive the same price p^*, while all the announcements below the threshold receive the announcement (there may be a corner solution with $\theta^* = \theta_L$, in which all sellers receive the same price regardless of the announcement). Note that, in equilibrium it cannot be the case that $p^* < \theta^*$. To see this, suppose for a moment that $p^* < \theta^*$. Then, all the dishonest sellers will announce θ^*, which implies that the sellers that announce $m > \theta^*$ are honest sellers who are telling the truth. But, this is a contradiction, because buyers are always willing to pay the expected quality conditional on each message, which implies that they will pay more than θ^* to all announcements above θ^*. Let $e(.)$ refer to the function that verbally encodes the message about type θ, where we take "θ" as the standard verbal representation in the natural language shared by the players of quality θ. Hence, in equilibrium $p^* \geq \theta^*$, and the equilibrium message function is given by:[10]

[9] Less informative equilibria in pure price strategies are not possible: since dishonest sellers overstate quality, in the low end of the quality spectrum only honest sellers who state their true quality remain. Hence, buyers will be willing to pay the announced quality, which implies a price lower than p^*.

[10] For $\theta(i) \in [\theta_L, \theta^*]$ and $h = 0$, "$\theta(i')$" $= e(\theta(i))$ is only possible when $\theta(i) = \theta^*$ and $p^* = \theta^*$. In all other cases, sellers strictly overstate quality.

$$m_S(\theta,h)=\begin{cases}\text{“}\theta\text{”}=e(\theta) & \text{if } \theta\in[\theta_L,\theta^*] \text{ and } h=\chi,\\ \text{“}\theta'\text{”}\in[\text{“}\theta^*\text{”},\text{“}\theta_H\text{”}]\geq e(\theta) & \text{if } \theta\in[\theta_L,\theta^*] \text{ and } h=0,\\ \text{“}\theta\text{”}=e(\theta) & \text{if } \theta\in(\theta^*,\theta_H].\end{cases}$$

Since buyers are willing to pay the expected quality conditional on each message, in equilibrium, we need $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$, where $\mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$ is the expected quality when a seller sends a message “m” > “θ^*” and the buyer offers p^*.

Summing up, the equilibrium conditions are given by: (i) $p^* \geq \theta^*$; (ii) $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$. In the Appendix we employ these conditions to determine p^* and θ^* (we also impose $p^* = \theta^*$ to maximize quality in the market for regular goods). The following proposition summarizes the results.

Proposition 4 *Consider the lemons model with a continuum of quality types uniformly distributed in the interval $[\theta_L, \theta_H]$, some honest sellers ($r < 1$) for whom the cost of misrepresentation is $\chi \geq \theta_H - \theta_L$, risk-neutral buyers willing to pay the expected quality of the product and $\bar{\alpha}_4 = \frac{\theta_H + \sqrt{r}\theta_L}{(1+\sqrt{r})\theta_H}$.[11] Then the perfect Bayesian equilibria within the set of pure price strategies are as follows when the efficiency condition $p^* = \theta^*$ is imposed:*

1. *If $\alpha \leq \bar{\alpha}_4$, there is a Pareto efficient and partially informative equilibrium if the traceability condition is satisfied. Let $\theta^* = (\theta_H + \sqrt{r}\theta_L)(1+\sqrt{r})^{-1}$. In this equilibrium, there are two categories of markets, where quality θ^* is the dividing line between both. The first category includes a continuum of lemons markets, with an individual market for each quality $\theta \in [\theta_L, \theta^*]$, where the price in each market “θ” is $p(\text{“}\theta\text{”}) = \theta$. The second category is a single market for regular goods, where qualities $\theta \in [\theta_L, \theta_H]$ are all traded at price $p^* = \theta^*$, given that expected quality is independent of announced quality “θ” $\in$ (“θ^*”, “θ_H”].*

2. *If $\alpha > \bar{\alpha}_4$, there is a Pareto inefficient and partially informative equilibrium if the traceability condition is satisfied. Let $\theta^* = \frac{\alpha\sqrt{r}}{\alpha(1+\sqrt{r})-1}\theta_L$. In this equilibrium, there are two categories of markets. The first category includes a continuum of lemons markets, with an individual market for each quality $\theta \in [\theta_L, \theta^*]$, where the price in each market “θ” is $p(\text{“}\theta\text{”}) = \theta$. The second category is a single market for regular goods, where qualities $\theta \in \left[\theta_L, \frac{\theta^*}{\alpha}\right]$ are all traded at price $p^* = \theta^*$, given that expected quality is independent of announced quality “θ” $\in$ (“θ^*”, “θ_H”]. If $\theta_L = 0$, there is full, instead of partial, market unraveling, and any message may be sent in equilibrium.*

Proof: See the Appendix. ■

Misrepresentation costs have two effects on equilibrium outcomes. First, comparing Proposition 2 Part 1 with Proposition 4 Part 1, misrepresentation costs allow to extend the region which supports a Pareto efficient allocation. In particular, note that $\bar{\alpha}_2 < \bar{\alpha}_4$. Second, comparing Proposition 2 Part 2 with Proposition 4 Part 2, even when $\alpha > \bar{\alpha}_4$ the equilibrium allocation with misrepresentation costs Pareto dominates the equilibrium allocation without misrepresentation costs. In particular, when there are no misrepresentation costs only qualities in the interval $\left[\theta_L, \frac{\theta_L}{2\alpha-1}\right]$ are traded, while with misrepresentation

[11] The requirement $\chi \geq \theta_H - \theta_L$ can be relaxed to $\chi \geq \frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}} - \theta_L$ to derive the results in Proposition 4. Otherwise, we revert to the results in Proposition 2.

costs qualities in the interval $\left[\frac{\theta_L}{2\alpha-1}, \frac{\sqrt{r}\theta_L}{\alpha(1+\sqrt{r})-1}\right]$ are also traded ($\alpha > \bar{\alpha}_4$ implies that this interval is non-empty). For $\theta_L = 0$, the proportion of honest sellers is too low to prevent complete market unraveling once $\alpha > \bar{\alpha}_4 = \frac{1}{(1+\sqrt{r})}$. The Pareto efficient equilibria in the range $(\bar{\alpha}_2, \bar{\alpha}_4]$, as well as those in the interval $\left[\frac{\theta_L}{2\alpha-1}, \frac{\sqrt{r}\theta_L}{\alpha(1+\sqrt{r})-1}\right]$ for $\alpha > \bar{\alpha}_4$, can again be described as trust equilibria.

5 Continuum of misrepresentation cost types

In this section we study the cases where there is a continuum of misrepresentation cost types ($H = [0, \chi]$). We first consider two quality types ($\Theta = \{\theta_L, \theta_H\}$), and then a continuum of quality types ($\Theta = [\theta_L, \theta_H]$).

5.1 Two quality types

Suppose that there are two types of quality $\theta_i \in \Theta = \{\theta_L, \theta_H\}$, and a continuum of honesty types $h \in H = [0, \chi]$, with $\chi > 0$. The probability a good is high quality is $q \in (0, 1)$, while honesty types are uniformly distributed with density $1/\chi$.

As before, let $m_S(\theta_i, h)(\text{“}m\text{”})$ denotes the probability that a seller of type (θ_i, h) sends message “m” $\in \{\text{“}\theta_L\text{”}, \text{“}\theta_H\text{”}\}$. For each message “$m$”, let $\mu(\text{“}m\text{”})(\theta_i)$ be the conditional probability that the product is type θ_i when message “m” is observed. Then, using Bayes’ rule

$$\mu(\text{“}m\text{”})(\theta_i) = \frac{\Pr(\theta_i) \int_h \frac{m_S(\theta_i,h)(\text{“}m\text{”})}{\chi} dh}{q \int_h \frac{m_S(\theta_L,h)(\text{“}m\text{”})}{\chi} dh + (1-q) \int_h \frac{m_S(\theta_H,h)(\text{“}m\text{”})}{\chi} dh},$$

unless “m” is not sent by any type, in which case the only requirement is that $\mu(\text{“}m\text{”})(\theta_i) \in [0, 1]$. Since sellers only accept price offers that cover their opportunity costs, let $\tilde{\mu}(\text{“}m\text{”}, p)(\theta_i)$ be the probability that the buyer effectively obtains a product of quality θ_i in response to message “m” by offering the price $p \in [\theta_L, \theta_H]$. Then:

$$\tilde{\mu}(\text{“}m\text{”}, p)(\theta_i) = \frac{a_S(\theta_i, p)(a) \Pr(\theta_i) \int_h \frac{m_S(\theta_i,h)(\text{“}m\text{”})}{\chi} dh}{a_S(\theta_L, p)(a)\, q \int_h \frac{m_S(\theta_L,h)(\text{“}m\text{”})}{\chi} dh + a_S(\theta_H, p)(a)(1-q) \int_h \frac{m_S(\theta_H,h)(\text{“}m\text{”})}{\chi} dh},$$

where $a_S(\theta_i, p)(a) = 1$ if $p \geq \alpha\theta_i$ and $a_S(\theta_i, p)(a) = 0$, otherwise. Given $\tilde{\mu}(\text{“}m\text{”}, p)(\theta_i)$, we can compute the expected quality for each possible message “m” and price p offered by the seller as follows:

$$\mathbf{E}_{\tilde{\mu}}[\theta \mid \text{“}m\text{”}, p] = \theta_L \tilde{\mu}(\text{“}m\text{”}, p)(\theta_L) + \theta_H \tilde{\mu}(\text{“}m\text{”}, p)(\theta_H).$$

Suppose that sellers use the following message function:

$$\begin{aligned} m_S(\theta_L, h)(\text{“}\theta_H\text{”}) &= 1 \text{ if } h \in [0, h^*), \\ m_S(\theta_L, h)(\text{“}\theta_L\text{”}) &= 1 \text{ if } h \in [h^*, \chi], \\ m_S(\theta_H, h)(\text{“}\theta_H\text{”}) &= 1 \text{ for all } h. \end{aligned}$$

Since only low quality types send message “θ_L”, a dominant strategy for the buyer is to offer $p_B(\text{“}\theta_L\text{”}) = \theta_L$. As to message “$\theta_H$”,the probability the buyer obtains a high quality product when observing message θ_H and offering price p is given by $\tilde{\mu}\left(\text{"}\theta_H\text{"}, p\right)\left(\theta_H\right) = \frac{a_S(\theta_H,p)(a)q}{a_S(\theta_H,p)(a)q+(1-q)\left(\frac{h^*}{\chi}\right)}$. Moreover, since buyers are willing to pay the expected quality, in equilibrium we have $p_B\left(\text{"}\theta_H\text{"}\right) = p^*$, where

$$p^* = \mathbf{E}_{\tilde{\mu}}\left[\theta \mid \text{"}\theta_H\text{"}, p^*\right] = \left[\frac{(1-q)\left(\frac{h^*}{\chi}\right)}{a_S\left(\theta_H, p^*\right)(a)\, q + (1-q)\left(\frac{h^*}{\chi}\right)}\right]\theta_L + \left[\frac{a_S\left(\theta_H, p^*\right)(a)\, q}{a_S\left(\theta_H, p^*\right)(a)\, q + (1-q)\left(\frac{h^*}{\chi}\right)}\right]\theta_H.$$

It only remains to see if sellers have an incentive to deviate. Clearly, a seller with a high quality product always prefer to tell the truth. A type h^* with a lemon must be indifferent between misrepresenting or not the truth. The cost of lying for a seller of type h is h; the benefit is the expected price gain from reporting "θ_H" instead of "θ_L". If the seller reports "θ_H", the price offer is p^*, while for report "θ_L" the offer is θ_L. Therefore

$$h^* = \min\left\{p^* - \theta_L, \chi\right\}.$$

Thus, a seller with a lemon and $h \geq h^*$ does not misrepresent quality, while a seller with a lemon and $h < h^*$ prefers to lie.

Summing up, the equilibrium conditions are given by: (i) $h^* = \min\left\{p^* - \theta_L, \chi\right\}$; and (ii) $p^* = \mathbf{E}_{\tilde{\mu}}\left[\theta \mid \text{"}\theta_H\text{"}, p^*\right]$. In the Appendix we employ these conditions to determine p^* and h^*. The following proposition summarizes the results.

Proposition 5 *Consider the lemons model with two quality types, θ_L and θ_H ($\theta_H > \alpha\theta_L$), a continuum of honesty types uniformly distributed in the interval $[0, \chi]$, with $\chi > 0$, risk-neutral buyers willing to pay the expected quality of the product and $\bar{q}_5 = \frac{(\alpha\theta_H - \theta_L)^2}{\theta_H(1-\alpha)\chi + (\alpha\theta_H - \theta_L)^2}$. Then the perfect Bayesian equilibria within the set of pure price strategies are as follows :*

1. *If $\chi \leq \alpha\theta_H - \theta_L$, misrepresentation costs are irrelevant and there are two cases. (i) If $q \geq \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}$, there is a Pareto efficient and uninformative ($h^* = \chi$) equilibrium. In this equilibrium there is only one market ("θ_H") in which both qualities are traded at price $p^* = \mathbf{E}_q[\theta]$. (ii) If $q < \bar{q}_1$, no equilibrium in pure price strategies exists.*

2. *If $\chi > \alpha\theta_H - \theta_L$, misrepresentation costs are relevant and there are three cases. (i) If $q \geq \frac{\chi}{\theta_H - \theta_L}$, there is a Pareto efficient and uninformative ($h^* = \chi$) equilibrium. In this equilibrium there is only one market ("θ_H") in which both qualities are traded at price $p^* = \mathbf{E}_q[\theta]$. (ii) If $\bar{q}_5 \leq q \leq \frac{\chi}{\theta_H - \theta_L}$ there is a unique Pareto efficient and partially informative ($h^* = p^* - \theta_L < \chi$) equilibrium if the traceability condition is satisfied. In this equilibrium there are two markets. In the lemons market ("θ_L") only goods of quality θ_L are trade at the price θ_L. In the market for regular goods ("θ_H"), both types of qualities are traded at the price $p^* = \theta_L + \frac{q\chi}{2(1-q)}\left[\sqrt{1 + \frac{4(1-q)(\theta_H - \theta_L)}{\chi q}} - 1\right]$. (iii) Finally if $q < \bar{q}_5$, no equilibrium in pure price strategies exists.*

Proof: see the Appendix. ■

Corollary 1 *Under the assumption of the previous proposition, let $q_{\min}$ ($\alpha_{\max}$) be the minimum q (maximum α) for which there is a Pareto efficient equilibrium. Then $q_{\min}$ and $\alpha_{\max}$ are given by:*

$$q_{\min} = \begin{cases} \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L} & \text{if } \chi \leq \alpha\theta_H - \theta_L, \\ \bar{q}_5 = \frac{(\alpha\theta_H - \theta_L)^2}{\theta_H(1-\alpha)\chi + (\alpha\theta_H - \theta_L)^2} & \text{if } \chi > \alpha\theta_H - \theta_L, \end{cases}$$

and

$$\alpha_{\max} = \begin{cases} \bar{\alpha}_1 = \frac{(1-q)\theta_L + q\theta_H}{\theta_H} & \text{if } \chi \leq q(\theta_H - \theta_L), \\ \bar{\alpha}_5 = \frac{\theta_L}{\theta_H} + \frac{q}{2(1-q)}\frac{\chi}{\theta_H}\left(\sqrt{1 + \frac{4(1-q)\left(1 - \frac{\theta_L}{\theta_H}\right)}{q\frac{\chi}{\theta_H}}} - 1\right) & \text{if } \chi > q(\theta_H - \theta_L), \end{cases}$$

respectively.

Proof: see the Appendix. ■

From Proposition 1 and Proposition 5 Part 1, when misrepresentation costs are inexistent, or low ($\chi \leq \alpha\theta_H - \theta_L$), it is only possible to support a Pareto efficient allocation if $q \geq \bar{q}_1$. Once costs of misrepresentation for some sellers are sufficiently high ($\chi > \alpha\theta_H - \theta_L$), by Proposition 5 Part 2 it is possible to support a Pareto efficient allocation if $q \geq \bar{q}_5$. Since $\bar{q}_1 > \bar{q}_5$ if and only if $\chi > \alpha\theta_H - \theta_L$, high misrepresentation costs extend the region for which we can support a Pareto efficient allocation and give rise to trust equilibria. The reason is that some sellers of low quality products are not willing to lie, which induces an increase in the expected quality of the good that sellers claim has a high quality, and hence in the price buyers are willing to pay. On the other hand, if $q < \bar{q}_5$, there is no price high enough for high quality good sellers to accept the deal, and low enough to induce an important fraction of lemons sellers to tell the truth, so the market for high quality products breaks down.

Corollary 1 shows that the maximum α for which it is possible to support a Pareto efficient allocation is a weakly increasing function of χ. Thus, higher misrepresentations costs never worsen the situation, in the sense that a higher value of $\frac{\chi}{\theta_H}$ always allows to support an efficient allocation for the same or higher values of α. Moreover, $\alpha_{\max}$ is a constant for $\chi \leq q(\theta_H - \theta_L)$ and a strictly concave function of $\frac{\chi}{\theta_H}$ for $\chi > q(\theta_H - \theta_L)$.

5.2 A continuum of quality types

Suppose that there is a continuum of quality types $\theta \in \Theta = [\theta_L, \theta_H]$, and a continuum of misrepresentation cost types $h \in H = [0, \chi]$, with $\chi > 0$. Quality is uniformly distributed with density $(\theta_H - \theta_L)^{-1}$, while misrepresentation cost types are uniformly distributed with density χ^{-1}.

We look for an equilibrium with a price strategy of the following form, since a less informative equilibrium is again not possible if some sellers honestly reveal low quality:

$$p_B(\text{“}m\text{”}) = \begin{cases} m & \text{if “}m\text{”} \in [\text{“}\theta_L\text{”}, \text{“}\theta^*\text{”}], \\ p^* & \text{if “}m\text{”} \in (\text{“}\theta^*\text{”}, \text{“}\theta_H\text{”}]. \end{cases}$$

Then, the message function is as follows, where $e(.)$ again refers to the function that encodes the message about type θ in natural language:

$$m_S(\theta,h) = \begin{cases} \text{“}\theta\text{”} = e(\theta) & \text{if } \theta \in [\theta_L, \theta^*] \text{ and } h \geq h^*(\theta), \\ \text{“}\theta'\text{”} \in (\text{“}\theta^*\text{”}, \text{“}\theta_H\text{”}] > e(\theta) & \text{if } \theta \in [\theta_L, \theta^*] \text{ and } h < h^*(\theta), \\ \text{“}\theta\text{”} = e(\theta) & \text{if } \theta \in (\theta^*, \theta_H]. \end{cases}$$

The fraction of sellers that belong to the quality range $[\theta_L, \theta^*]$ is $q_L = \Pr(\theta \in [\theta_L, \theta^*]) = \frac{\theta^* - \theta_L}{\theta_H - \theta_L}$, while the fraction that belong to the quality range $(\theta^*, \theta_H]$ is $q_H = \Pr(\theta \in (\theta^*, \theta_H]) = \frac{\theta_H - \theta^*}{\theta_H - \theta_L}$. The expected qualities in each range are $\mathbf{E}(\theta \mid \theta \in [\theta_L, \theta^*]) = \frac{\theta^* + \theta_L}{2}$ and $\mathbf{E}(\theta \mid \theta \in (\theta^*, \theta_H]) = \frac{\theta^* + \theta_H}{2}$, respectively. However, given the message function, some sellers in the range $[\theta_L, \theta^*]$ prefer to lie. Indeed, for each $\theta(i) \in [\theta_L, \theta^*]$, the proportion of sellers that are willing to misrepresent quality is implicitly given by:

$$h^*(\theta) = \min\{p^* - \theta, \chi\}.$$

From this equation we can compute the fraction of sellers that misrepresent quality $q_{MIS} = \Pr(\theta \in [\theta_L, \theta^*], \text{“}m\text{”} > \text{“}\theta^*\text{”})$, and the average quality of those that misrepresent $\mathbf{E}_{MIS} = \mathbf{E}(\theta \mid \theta \in [\theta_L, \theta^*], \text{“}m\text{”} > \text{“}\theta^*\text{”})$. Then, we can use q_{MIS} and $\mathbf{E}_{MIS}$ in order to compute $\mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p]$, the average quality of the sellers that report “m” > “θ^*” when buyers offer p. Finally, since buyers are willing to pay the expected quality, in equilibrium, $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$.

Summing up, the equilibrium conditions are given by: (i) $h^*(\theta) = \min\{p^* - \theta, \chi\}$; and (ii) $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$. In the Appendix we employ these conditions to determine p^* and $h^*(\theta)$. Following the procedure in Proposition 4, we impose the condition $p^* = \theta^*$ so quality is maximized in the market for regular goods. The following proposition summarizes the results.

Proposition 6 *Consider the lemons model with a continuum of quality types uniformly distributed in the interval $\Theta = [\theta_L, \theta_H]$, a continuum of misrepresentation cost types uniformly distributed in the interval $H = [0, \chi]$, with $\chi > 0$, risk-neutral buyers willing to pay the expected quality of the product, and $\bar{\alpha}_6 = \frac{(\sqrt{6}-2)\theta_L + (3-\sqrt{6})\theta_H}{\theta_H}$. Then the perfect Bayesian equilibria within the set of pure price strategies are as follows when the traceability condition is satisfied, and the efficiency condition $p^* = \theta^*$ is imposed:*

1. *For $\alpha \leq \bar{\alpha}_6$, there is a Pareto efficient and partially informative equilibrium with two categories of markets if $\frac{\chi}{\theta_H} \geq \sqrt{3\left(1 - \frac{\theta_L}{\theta_H}\right)\left(2\alpha - 1 - \frac{\theta_L}{\theta_H}\right)}$. The first category is a continuum of lemons markets with an individual market for each product of quality $\theta \in [\max\{\theta_L, \theta^* - \chi\}, \theta^*]$, where the price in market “θ” is $p(\text{“}\theta\text{”}) = \theta$. The second category is a single market for regular goods in which qualities of any type $\theta \in [\theta_L, \theta_H]$, announced as qualities “θ” $\in$ (“θ^*”, “θ_H”], are traded either at price $\frac{p^*}{\theta_H} = \frac{1}{6}\left(\frac{\chi}{\theta_H}\right)^2 + \frac{1}{2}$, if $\frac{\chi}{\theta_H} \leq (3 - \sqrt{6})\left(1 - \frac{\theta_L}{\theta_H}\right)$, or at the price p^* that is the unique solution of $Q\left(\frac{p^*}{\theta_H}\right) = 0$, if $\frac{\chi}{\theta_H} \geq (3 - \sqrt{6})\left(1 - \frac{\theta_L}{\theta_H}\right)$.*

2. *If $\alpha \geq \bar{\alpha}_6$, there is a Pareto efficient and partially informative equilibrium with two categories of markets if $\frac{\chi}{\theta_H} \geq \frac{2}{3}\frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$. The first category is a continuum of lemons markets with an individual market for each product of quality $\theta \in [\max\{\theta_L, \theta^* - \chi\}, \theta^*]$, where the price in each market “θ” is $p(\text{“}\theta\text{”}) = \theta$. The second category is a single market for regular goods in which qualities of any type $\theta \in [\theta_L, \theta_H]$, announced as qualities “θ” $\in$ (“θ^*”, “θ_H”], are traded at the price p^* that is the unique solution of $Q\left(\frac{p^*}{\theta_H}\right) = 0$.*

3. If $\frac{\chi}{\theta_H} < \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha-1-\frac{\theta_L}{\theta_H}\right)}$ for $\alpha \le \bar{\alpha}_6$, or $\frac{\chi}{\theta_H} < \frac{2}{3}\frac{\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ for $\alpha \ge \bar{\alpha}_6$, there is no Pareto efficient equilibrium. There is, however, a Pareto inefficient partially informative equilibrium with two categories of markets. The first category is a continuum of lemons markets with an individual market for each product of quality $\theta \in [\max\{\theta_L, \theta^* - \chi\}, \theta^*]$, where the price in each market "θ" is $p(\text{"}\theta\text{"}) = \theta$. The second category is a single market for regular goods in which qualities of any type $\theta \in \left[\theta_L, \bar{\theta}_H\right]$ with $\bar{\theta}_H < \theta_H$, announced as qualities "θ" $\in$ ("θ^*", "θ_H"], are traded.[12]

Proof: See the Appendix. ■

Corollary 2 *Under the assumptions of the previous proposition, let $\alpha_{\max}$ be the function that gives the maximum value of α for each value of $\frac{\chi}{\theta_H}$ such that there is a Pareto efficient equilibrium. Then, $\alpha_{\max}$ is given by:*

$$\alpha_{\max} = \begin{cases} \frac{\left(\frac{\chi}{\theta_H}\right)^2}{6\left(1-\frac{\theta_L}{\theta_H}\right)} + \frac{1}{2}\left(1+\frac{\theta_L}{\theta_H}\right), & \text{for } 0 < \frac{\chi}{\theta_H} \le \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right), \\ f^{-1}\left(\frac{\chi}{\theta_H}\right), & \text{for } \frac{\chi}{\theta_H} \ge \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right). \end{cases}$$

where f^{-1} is the inverse of $f(\alpha) = \frac{2}{3}\frac{\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ for $\alpha \in [3-\sqrt{6}+(\sqrt{6}-2)\frac{\theta_L}{\theta_H}, 1)$. Moreover, $\alpha_{\max}$ is a continuous and increasing function of $\frac{\chi}{\theta_H}$, which is strictly convex for $\frac{\chi}{\theta_H} < \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$ and strictly concave for $\frac{\chi}{\theta_H} > \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$.

Proof: See the Appendix. ■

Since $\bar{\alpha}_2 < \bar{\alpha}_6$, the presence of honest agents gives rise to Pareto efficient equilibria in a range that was impossible with dishonest agents in Proposition 2; in other words, trust equilibria in Proposition 6 again enlarge the set where trade is possible. Furthermore, with a double continuum there is never full market unraveling, and hence no babbling equilibria, as in Proposition 4. Corollary 2 characterizes how effective misrepresentations costs are to overcome market breakdown. Since $\alpha_{\max}$ is an increasing function of $\frac{\chi}{\theta_H}$, higher misrepresentations costs always improve the situation, because a higher value of $\frac{\chi}{\theta_H}$ allows to support an efficient allocation for higher values of α. Since $\alpha_{\max}$ is strictly convex for $\frac{\chi}{\theta_H} < \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$ and strictly concave for $\frac{\chi}{\theta_H} > \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$, misrepresentation costs initially have increasing returns, in the sense that successive increases in misrepresentation costs induce positive and increasing changes in $\alpha_{\max}$ and therefore, in the region in which can support a Pareto efficient allocation. However, eventually misrepresentation costs enter a stage of decreasing returns, and further increases induce positive but decreasing changes in $\alpha_{\max}$.

6 Mixed price strategies

Up to now we have restricted the equilibria to pure price strategies. When a Pareto efficient allocation can be supported with a pure price strategy equilibrium, it is not very interesting to search for possible

[12] In the Appendix we deduce $\bar{\theta}_H$ and the price p^* at which qualities announced as "θ" $\in$ ("θ^*", "θ_H"] are traded.

equilibria in mixed strategies. However, when this is not the case, or when it is not possible to find an equilibrium in pure price strategies (as in the setup of Proposition 5), it is natural to ask what would happen if we considered mixed price strategies. Hence, in this section we study if mixed price strategy equilibrium help to complete markets.

6.1 Two qualities

We begin considering if mixed price strategy equilibria are possible in the settings of Propositions 1, 3, and 5.

It turns out that a mixed price strategy equilibrium is not possible in the setting of the Pareto inefficient equilibrium of Proposition 1 where $q < \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}$, and all sellers are dishonest ($h = 0$) and perfectly willing to lie. To see this, consider what happens if sellers of lemons must decide between either announcing quality "θ_L", and receiving price θ_L with certainty, or announcing quality "θ_H" and receiving the following bundle: either a high price ($p^* \geq \alpha\theta_H$) with probability $1 - \sigma^*$, or θ_L with probability σ^*.[13] To be indifferent between both strategies, the following no arbitrage condition between markets "θ_L" and "θ_H" must hold for the sellers of lemons:

$$\theta_L - \alpha\theta_L = (1 - \sigma^*)(p^* - \alpha\theta_L) + \sigma^*(\theta_L - \alpha\theta_L). \tag{3}$$

This condition, alas, is never satisfied, because it implies $(1 - \sigma^*)(\theta_L - \alpha\theta_L) = (1 - \sigma^*)(p^* - \alpha\theta_L)$, which requires either $\sigma^* = 1$, i.e., no trade in market "θ_H", or $p^* = \theta_L$, which also implies that no high-quality products are traded because the reservation utility is not covered. Hence, only the pure strategy price equilibria described in Proposition 1 exist.

This same logic applies to the Pareto inefficient equilibrium of Proposition 3 where $q < \bar{q}_3 = \frac{r(\alpha\theta_H - \theta_L)}{(1-\alpha)\theta_H + r(\alpha\theta_H - \theta_L)}$ and there are both honest ($h = \chi$) and dishonest ($h = 0$) sellers, since the marginal seller in the case of partial market breakdown is an utterly dishonest seller with $h = 0$. Hence the relevant condition is again (3), which is only satisfied for either $\sigma^* = 1$, or $p^* = \theta_L$.

As to the setting of Proposition 5 where either $q < \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}$ for $\chi \leq \alpha\theta_H - \theta_L$, or $q < \bar{q}_5 = \frac{(\alpha\theta_H - \theta_L)^2}{\theta_H(1-\alpha)\chi + (\alpha\theta_H - \theta_L)^2}$ for $\chi > \alpha\theta_H - \theta_L$, we found there is no equilibrium in pure price strategies because if every seller states it has quality θ_H, buyers are not willing to trade, so sellers of lemons do not want to lie, while if every seller states its true quality, buyers are willing to pay it, so seller of lemons are willing to lie. Unlike the prior setups, since there is a continuum of misrepresentation cost types, the marginal seller is some type $h^* > 0$. The no arbitrage condition is here given by:

$$\theta_L - \alpha\theta_L = (1 - \sigma^*)(p^* - \alpha\theta_L) + \sigma^*(\theta_L - \alpha\theta_L) - h^* \Longrightarrow h^* = \min\{(1 - \sigma^*)(p^* - \theta_L), \chi\}. \tag{4}$$

More formally, let sellers send the following messages: $m_S(\theta_H, h)(\text{“}\theta_H\text{”}) = 1$ for all h, $m_S(\theta_L, h)(\text{“}\theta_H\text{”}) = 1$ for $h \in [0, h^*)$ and $m_S(\theta_L, h)(\text{“}\theta_L\text{”}) = 1$ for $h \in [h^*, \chi]$. Let buyers offer the following price function: $p_B(\text{“}\theta_L\text{”})(\theta_L) = 1$, $p_B(\text{“}\theta_H\text{”})(\theta_L) = \sigma^*$, and $p_B(\text{“}\theta_H\text{”})(p^*) = 1 - \sigma^*$, where $p^* \in [\alpha\theta_H, \theta_H]$ and $\sigma^* \in (0, 1)$. Moreover, sellers use the following acceptance rule: $a_S(\theta_H, p)(a) = 1$ if $p \geq \alpha\theta_H$, $a_S(\theta_H, p)(a) = 0$

[13] Equivalently, we can assume that with probability σ^* there is no transaction, in which case a seller with a lemon can return to market "θ_L" and make a sale at price θ_L.

if $p < \alpha\theta_H$, and $a_S(\theta_L, p)(a) = 1$, and buyers are always willing to pay the expected quality, i.e., in equilibrium

$$p^* = \left[\frac{(1-q)\left(\frac{h^*}{\chi}\right)}{(1-q)\left(\frac{h^*}{\chi}\right)+q}\right]\theta_L + \left[\frac{q}{(1-q)\left(\frac{h^*}{\chi}\right)+q}\right]\theta_H. \tag{5}$$

Since $p^* \in (\theta_L, \theta_H]$ and $\sigma^* \in (0,1)$, from (4), it must be the case that some sellers of lemons will lie ($h^* > 0$). Thus, we only need to consider possible cases: when $h^* = \chi$, and when $h^* < \chi$. Suppose that $h^* = \chi$, then all sellers of lemons misrepresent quality and, hence, $p^* = (1-q)\theta_L + q\theta_H$. However, we are in a region for which $q < \bar{q}_1$, which implies that $p^* = (1-q)\theta_L + q\theta_H < \alpha\theta_H$. Next, suppose that $h^* < \chi$, then by (4), $h^* = (1-\sigma^*)(p^* - \theta_L)$. Using this equality and (5) leads to $1 - \sigma^* = \frac{\chi q(\theta_H - p^*)}{(1-q)(p^*-\theta_L)^2}$ and $h^* = \frac{\chi q(\theta_H - p^*)}{(1-q)(p^*-\theta_L)}$. This implies that $p^* \in [\alpha\theta_H, \theta_H)$ and $1-\sigma^* \in \left(0, \frac{\chi q(1-\alpha)\theta_H}{(1-q)(\alpha\theta_H - \theta_L)^2}\right]$, with $1-\sigma^*$ a decreasing function of p^*. Finally, note that $\frac{\chi q(1-\alpha)\theta_H}{(1-q)(\alpha\theta_H-\theta_L)^2} < 1$ if and only if $q < \bar{q}_5 = \frac{(\alpha\theta_H - \theta_L)^2}{\theta_H(1-\alpha)\chi + (\alpha\theta_H-\theta_L)^2}$. Thus, a range of equilibria are possible for each value of χ.

The following proposition summarizes the results.

Proposition 7 *Consider the lemons model with two quality types, θ_L and θ_H ($\theta_H > \alpha\theta_L$), and risk-neutral buyers willing to pay the expected quality of the product, in the regions where no Pareto efficient pure price strategy equilibrium exists.*[14]

1. *For utterly dishonest sellers ($H = \varnothing$), there is no equilibrium in mixed price strategies for $q < \bar{q}_1 = \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L}$.*

2. *For two honesty types ($H = \{0, \chi\}$), there is no equilibrium in mixed price strategies for $q < \bar{q}_3 = \frac{r(\alpha\theta_H-\theta_L)}{(1-\alpha)\theta_H + r(\alpha\theta_H - \theta_L)}$.*

3. *For a continuum of honesty types uniformly distributed in the interval ($H = [0, \chi]$), there is a Pareto inefficient partially informative equilibrium in mixed price strategies if either $q < \bar{q}_1 = \frac{\alpha\theta_H-\theta_L}{\theta_H-\theta_L}$, for $\chi \leq \alpha\theta_H - \theta_L$, or $q < \bar{q}_5 = \frac{(\alpha\theta_H-\theta_L)^2}{\theta_H(1-\alpha)\chi+(\alpha\theta_H-\theta_L)^2}$ for $\chi > \alpha\theta_H - \theta_L$, and the traceability condition holds. Sellers of lemons with $h < h^*$ misrepresent quality stating they have quality "θ_H". In market "θ_L" buyers offer θ_L with probability 1, while in market "θ_H" buyers offer θ_L with probability $\sigma^* \in (0,1)$ and p^* with probability $(1-\sigma^*)$. Thus, with probability $\sigma^* \in (0,1)$, products with quality θ_H are not traded.*

This proposition assumes that there are no transaction costs. If there were transactions costs of entering into the market, then a sufficiently high probability of market disappearance σ^* would push high quality products out of the market anyway. In other words, if owners of high-quality products can foresee that the probability of doing a sale is low enough, they will prefer not to enter the market in the first place.

[14] When we say "mixed price strategy" we refer to a non-degenerate mixed price strategy in which several prices (indeed two prices) are played with positive probability.

6.2 A continuum of qualities

We have seen that mixed price strategies help avoid total market disappearance of the high quality good in the discrete case with two qualities only if there is an interior solution where the marginal sellers have a dislike for misrepresenting quality. We now apply an analogous analysis to the settings of Propositions 2, 4, and 6.

In the case of the Pareto inefficient equilibria in Proposition 2, suppose price offers are augmented as follows. For announced qualities "m" in the interval $[$"θ_L", "$\frac{\theta^*}{\alpha}$"$]$, a price $p^* = \theta^* = \frac{\alpha\theta_L}{2\alpha-1}$ is offered. For messages "m" above "$\frac{\theta^*}{\alpha}$", with probability $\sigma($"m"$)$ a price p^* is offered, while with probability $1-\sigma($"m"$)$ a price $p($"m"$)$ is offered. Note that this mixed price strategy has the same structure as the one we employed for two quality types. That is, with probability $\sigma($"m"$)$, the price p^* targets the lower segment $[$"θ_L", "$\frac{\theta^*}{\alpha}$"$]$, while with probability $1-\sigma($"m"$)$, the price $p($"m"$)$ targets a seller with a good of quality $\theta > \frac{\theta^*}{\alpha}$ that tells the truth, i.e. "m" = "θ".

Sellers are willing to report the truth if the following no arbitrage condition holds:

$$p^* - \alpha\theta = [1-\sigma(\text{“}\theta\text{”})]\,[p(\text{“}\theta\text{”}) - \alpha\theta] + \sigma(\text{“}\theta\text{”})\,[p^* - \alpha\theta] \text{ for all } \theta. \tag{6}$$

However, this condition can only be satisfied if $p($"θ"$) = p^*$ for all θ or $\sigma($"θ"$) = 1$ for all θ, i.e., no equilibrium in mixed price strategies exists.

A similar argument can be applied to the Pareto inefficient equilibrium of Proposition 4, since the marginal seller again is a seller with $h = 0$. In Proposition 4, when the traceability condition applies, under partial market breakdown there was full revelation of some of the lowest qualities, while in an intermediate quality range goods where sold at a constant price p^* equal to expected quality $\theta^* = \alpha\frac{\sqrt{r}\theta_L}{(1+\sqrt{r})\alpha-1}$, and the highest qualities were out of the market. Suppose now that buyers pay whatever the sellers report if the report is lower than "θ^*", a fixed price $p^* = \theta^*$ if "m" is between "θ^*" and "$\frac{\theta^*}{\alpha}$", but a mixed strategy which offers p^* with probability $\sigma($"m"$)$ and $p($"m"$) > \frac{\theta^*}{\alpha}$ with probability $1-\sigma($"m"$)$ if "m" > "$\frac{\theta^*}{\alpha}$". Again, dishonest sellers are willing to report the truth if (6), which is only satisfied if $p($"θ"$) = p^*$ for all θ or $\sigma($"θ"$) = 1$ for all θ.

For the Pareto inefficient equilibrium of Proposition 6, suppose that buyers pay whatever the sellers report if "θ" is lower than "θ^*", a fixed price $p^* = \theta^*$ equal to expected quality if "θ" is between "θ^*" and "$\bar{\theta}_H$", and a mixed strategy which offers p^* with probability $\sigma($"m"$)$, and $p($"m"$)$ with probability $1-\sigma($"m"$)$, if "θ" > "$\bar{\theta}_H$". The argument here is a bit more complicated that in Proposition 4 because the marginal seller has $h^* > 0$. However, for the marginal sellers that have already incurred a misrepresentation cost, they are in the same situation as the marginal seller of Proposition 4: they lose nothing by announcing a quality higher than "$\bar{\theta}$", and returning to market ("θ^*", "$\bar{\theta}$"$]$ in case a sale is not closed. The difference with the setup of Proposition 5, where there is also a continuum of misrepresentation cost types, is that there the misrepresentation cost must be considered ex-ante. Here, in any equilibrium where some sellers already incur misrepresentation costs, the additional cost of exaggerating quality even more is zero in our setup.

The following proposition summarizes the results.

Proposition 8 *Consider the lemons model with a continuum of quality types, uniformly distributed in the interval $\Theta = [\theta_L, \theta_H]$, and risk-neutral buyers willing to pay the expected quality of the product, in the regions where no Pareto efficient pure price strategy exists.*

1. *For utterly dishonest sellers ($H = \varnothing$), there is no equilibrium in mixed price strategies for* $\alpha > \bar{\alpha}_2 = \frac{\theta_H+\theta_L}{2\theta_H}$.

2. *With two honesty types ($H = \{0, \chi\}$), there is no equilibrium in mixed price strategies for* $\alpha > \bar{\alpha}_4 = \frac{\theta_H+\sqrt{r}\theta_L}{(1+\sqrt{r})\theta_H}$.

3. *With a continuum of honesty types uniformly distributed on the interval ($H = [0, \chi]$), there is no equilibrium in mixed price strategies for either* $\frac{\chi}{\theta_H} < \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha-1-\frac{\theta_L}{\theta_H}\right)}$ *and* $\alpha \leq \bar{\alpha}_6$*, or* $\frac{\chi}{\theta_H} < \frac{2}{3}\frac{\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ *and* $\alpha \geq \bar{\alpha}_6$.

Hence, the results of Propositions 1 to 4 and 6 are without loss of generality. When there is no Pareto efficient equilibrium, there is no equilibrium in mixed price strategies either. Only when there are two quality types and a continuum of misrepresentation costs, mixed strategies make a difference. Indeed, in the setup of Proposition 5, when there is no Pareto efficient equilibrium, there is no pure price strategy equilibrium, but there exists a mixed price strategy equilibrium.

7 Symbolic signals

Though moral codes may be important, we must not ignore the key role of linguistic conventions themselves which we learn as we are brought up. Smith (1776) gives our ability to speak and communicate through natural language a key role in explaining human cooperation through the division of labor and exchange (Ángel Alonso-Cortés 2008). More fundamentally, moral codes like misrepresentation costs presuppose a conventional way to verbally represent the truth, otherwise neither true nor deceitful messages would be available. This conventional feature which is specific to natural language can be described resorting to the categories in semiotics, allowing us to make explicit a hidden assumption in all costly talk models.

In semiotics, the central idea is that of a sign that can convey information. Three main types of signs are distinguished: symbols, icons and indices (Daniel Chandler 1994). Symbols are characterized by being purely conventional: words are instances of symbols. Icons resemble their subject matter: caricatures, and road signs that show a winding line, are instances of icons. Indices are physically linked to some antecedent cause: smoke, footprints or medical symptoms are all examples of indices. As Chandler (1994) points out, this three-fold distinction, introduced by Charles S. Peirce as the most fundamental division of signs, does not separate signs into mutually exclusive categories. Icons, for instance, may have a symbolic component (and so may indices, as we now show). Rather, it captures, in decreasing order of conventionality, differing modes of relationship between signs and the information they convey.

The category of signals used in economics can be seen as an index that is voluntarily chosen by the sender to reveal either some underlying characteristic, or something about the sender's state of mind. For example, education helps to reveal productivity in the job market (Michael Spence 1973), guarantees, quality in goods markets (Akerlof 1970). Once we introduce misrepresentation costs, words also become signals. Besides costly talk, we have come across two signals they are legally binding: a firm offer by the buyer to pay a certain price, and a firm offer by the seller to transfer its property rights. On the other hand, the ownership record of the good in question, which is key for the traceability condition, is not a signal but rather an involuntary index, which is a by-product of market transactions.

Misrepresentation costs lead to a special kind of signal, what may be called "symbolic signals". The other voluntary and involuntary indices we consider, which provide hard evidence on what price the buyer will actually pay, on the willingness of the legal owner to transfer the good in question, and on whether the seller is the original owner or an intermediary, have a symbolic component as well, since they are all spelled out in natural language.

The symbolic character of costly talk has to do with the fact that talk is purely conventional. Michael Rescorla (2010) cites Nelson Goodman (1989: 80) on convention having two sides: on the one hand, the conventional is the ordinary, as against the novel; on the other hand, the conventional is the artificial, as against the natural. Though social conventions like language are in a sense artificial, and hence arbitrary because other conventions might do just as well, in another sense they are not, which explains why they represent the ordinary and usual: a language is a shared social convention in which we are born and raised. Hence, from the point of view of players that share a common language, the messages that may be used to communicate cannot be invented at will but must conform to the linguistic rules of that community. This feature appears in (2), where there is a cost of misrepresentation h if "m" $\neq$ "θ", i.e., if the sender uses a message that does not reflect the socially accepted way of verbally representing quality θ. More than the symbolic feature of language is at stake, else we would remain in the domain of cheap talk where the seller is indifferent between saying any two phrases (though this indifference might be broken by the beliefs of the receiver in equilibrium, see Streb and Torrens 2011).

Linguistic symbols are composed of three elements (Chandler 1994):

(i) the signifier or sign vehicle, a sequence of letters or sounds "w", e.g., "This car is a lemon.";

(ii) the signified, intension, or connotation "$\widehat{w}$", the concept that appears in our mind when we read or hear the signifier, e.g., a lousy car; and

(iii) the referent, extension, or denotation, the actual object w a signifier refers to, e.g., the car they are trying to sell us.

Though the signifier "w" is only a part of the whole, it is also customary to refer to the signifier as the symbol. Ferdinand de Saussure uses a dyadic model composed of signifier and signified, but Charles S. Peirce proposes instead a triadic model that is closer to this modern representation. However, both perspectives are useful to distinguish between language at two levels, as a convention and as a means of communication (Streb and Torrens 2011).

As a set of linguistic conventions, the signified is essential for words to convey meaning. Language can be seen as the set "W" comprised of all the messages "w" that can potentially be formed to utter comprehensible statements in a natural language shared by the speakers, describing it through de Saussure's dyadic model of linguistic symbols, i.e., signifier "w" and signified "$\widehat{w}$". As long as messages stick to the shared conventions in "W", they may be anything, including of course mere fiction instead of hard facts.

As a means of communication, Joseph Farrell's (1993: 515) remark on the distinction between comprehensibility and credibility of messages allows to distinguish two steps in the process of using a shared language to communicate meaning in concrete situations. First, the message must be correctly constructed according to the conventions in that language to be comprehensible. Speakers do not randomly use any word in the dictionary to name something. Rather, when there is asymmetric information, they rely on ordinary words to convey meaning to the listener. Given that, the following step is the credibility of the message, i.e., whether the statement implied by the message is true or not. Here the point of view of Peirce and earlier logicians comes to the fore: the referent of the message is crucial. This can be represented as follows. When the message is encoded, the sender thinks of a signifier "w_S" suited to the referent w_S, word-to-fit-world described by the encoding function $e(.)$; when the message is decoded, the receiver thinks of a referent w_R suited to the signifier "w_R", world-to-fit-word described by the decoding function $d(.)$:

$$\begin{aligned} \text{“}w_S\text{”} &= e(w_S), \\ w_R &= d(\text{“}w_R\text{”}). \end{aligned}$$

We assume that the message that is heard by the receiver ("m_R") coincides with the message uttered by the sender ("m_S"), with "m_R" = "m_S", which implies there is no noise in the communication. We rule out errors of perception by either sender or receiver: in the encoding stage, the signifier "w_S", the signified "$\widehat{w}_S$", and the referent w_S cohere, and the same holds for the decoding stage. Communication is fully effective when the starting and ending points coincide, $w_S = w_R$. In our examples, the referent w_S is a type θ_S, but in other setups it could also be an action, or the intention of an action. The issue we analyze here is that not all the information may be revealed due to wilful distortions of the sender, i.e., "m_S" $\neq$ "w_S".

We have a two-valued logic where the sender may be truthful ($T = 1$) or not ($T = 0$). We do not distinguish between degrees of falsehood, which in our examples would be by how much the sender distorts reported quality. However, since the sender may play mixed strategies, any degree of informativeness which falls in between the extremes of fully informative and outrightly misleading messages is potentially possible:

$$\begin{aligned} T(\text{“}m_S\text{”}) = 1 \;&\Leftrightarrow\; \text{“}m_S\text{”} = \text{“}w_S\text{”}, \\ T(\text{“}m_S\text{”}) = 0 \;&\Leftrightarrow\; \text{“}m_S\text{”} \neq \text{“}w_S\text{”}. \end{aligned}$$

Our approach is quite close to Demichelis and Weibull (2008: 1293), who propose a "meaning correspondence" that relies on a common language, to which they associate an honesty code. Demichelis and Weibull (2008: 1296) define the meaning correspondence as a mapping from a message (or a pair of messages, in the case of statements conditional on what the other player says) to a set of actions, which may be a singleton set. The message is true if the action that is carried out belongs to the announced set of actions. No honesty cost is incurred only if what is actually carried out belongs to the announced set of actions. Their approach also separates analytically the issue of an agreed meaning of a statement (what we call "language as a shared convention") from that of its truth-value in a concrete situation (what we call "language as a means of communication"), which is what is at stake in the meaning correspondence of Demichelis and Weibull. The honesty code is an altogether independent issue, not only because this code depends on the prior existence of a linguistic convention on what statement is true or not in a given situation, but also because a shared language might be a useful coordinating device even without any honest agents (Streb and Torrens 2011). The view in Demichelis and Weibull (2008) can be generalized, as we do here, since what is communicated might not be only an intended action but rather a type, as in the market for lemons. Furthermore, our analysis puts the issue the other way around: given some type (alternatively, some intended action), the sender communicates verbally something that may coincide or not with the actual type (alternatively, the intended action). In our examples we always consider single-valued messages, so truth depends on whether the message coincides or not with the underlying quality.

8 Final remarks

We analyze the influence of introducing sellers with misrepresentation costs on the equilibrium of the market for lemons, where there is asymmetric information on quality. We first consider two, and then a continuum, of types that differ in misrepresentation costs, in either a setup with two, or a continuum, of qualities. In our model, honesty is endogenous, depending both on character (misrepresentation costs) and market opportunities.

Our main result for the market for lemons is that the existence in equilibrium of honest agents that mean what they say is not enough to make trade more likely, unless a traceability condition is satisfied which prevents dishonest sellers from doing arbitrage between the lemons markets and the regular market. This traceability condition is satisfied if an ownership record exists that allows buyers to distinguish between ordinary owners who sell lemons with probability lower than one, and intermediaries who sell lemons with probability one. Given that, equilibria in which high-quality goods are traded with positive probability arise, which would not be possible if all agents were dishonest; these equilibria can be described as trust equilibria.

When there are only two types of misrepresentation costs, we find that the presence of honest sellers does not suffice to eliminate uninformative equilibria. In the case of two quality types, there are uninformative equilibria if the proportion of honest sellers in the market is too low. In the case of a continuum of quality types, there are also uninformative equilibria in a special case, when the lowest quality is zero, and the proportion of honest sellers in the market is too low. This differs from the results in Callander and Wilkie's (2007) setting, where a continuum of policy preferences and two types of misrepresentation costs always allow to eliminate uninformative equilibria.

We also consider a continuum of types of misrepresentation costs, which to the best of our knowledge has not been studied before. Honest sellers are determined endogenously as those whose misrepresentation costs are above some cutpoint. With two quality types, misrepresentation costs are not enough to eliminate uninformative equilibria, but if there is an uninformative equilibrium it is unique in that all sellers of lemons strictly prefer to overstate quality. Moreover, since agents refrain from lying in equilibrium if it does not pay off, equilibria in pure price strategies might not exist if there is partial market breakdown. However, we find that a mixed price strategy equilibrium exists where the disappearance of the highest quality is strictly positive, but smaller than one. With a continuum of quality types, the novelty is that there never exists full market unraveling.

Going beyond the bounds of the market for lemons, with misrepresentation costs talk is no longer cheap but rather a costly signal, as in Callander and Wilkie (2007) and Kartik, Ottaviani, and Squintani (2007). Costly talk differs from other types of signals, however, because the communication process is mediated by a special kind of signs that have a purely conventional character, namely symbols that belong to a language common to all the players. Costly talk is hence a "symbolic signal" that presupposes linguistic conventions, otherwise the truth and falsehood of the statement uttered are undefined.

In semiotics, indices are any type of sign that is the consequence of some cause, like smoke, of fire, or fever, of a flu. Signals are a particular type of indices, namely indices that are voluntarily chosen by the sender. As we have seen for costly talk, however, indices can have a symbolic component. Paying money and transfering title deeds are two other symbolic indices — and, more specifically, symbolic signals — we encountered, which underpin the functioning of markets, and presuppose a state and the rule of law. The ownership record, which allows to avoid arbitrage between markets if it is available to buyers, is also a symbolic index, though it is not chosen voluntarily but is rather a by-product of market transactions.

References

[1] Akerlof, George (1970), "The market for 'lemons': Quality uncertainty and the market mechanism", *Quarterly Journal of Economics* 84: 488–500.

[2] Alonso-Cortés, Ángel (2008), "Trade and language: Adam Smith's rhetoric of persuasion", mimeo, Universidad Complutense de Madrid.

[3] Blanton, Brad (1996), *Radical honesty. How to transform your life by telling the truth*, New York, NY: Dell Publishing.

[4] Callander, Steven, and Simon Wilkie (2007), "Lies, damned lies, and political campaigns", *Games and Economic Behavior* 60: 262–286.

[5] Chandler, Daniel (1994), *Semiotics for Beginners,* http://www.aber.ac.uk/media/Documents/S4B/ [accessed 13.Jan.10].

[6] Crawford, Vincent P., and Joel Sobel (1982), "Strategic information transmission", *Econometrica* 50: 1431–1451.

[7] Coase, Ronald H. (1976), "Adam Smith's view of man", *Journal of Law and Economics* 19, 529–546.

[8] Demichelis, Stefano, and Jörgen W. Weibull (2008), "Language, meaning and games: A model of communication, coordination and evolution", *American Economic Review* 98: 1292–1311.

[9] Farrell, Joseph (1993), "Meaning and credibility in cheap-talk games", *Games and Economic Behavior* 5, 514–531.

[10] Gibbons, Robert (1992), *Game theory for applied economists*, Princeton, NJ: Princeton University Press.

[11] Goodman, Nelson (1989), "Just the Facts, Ma'am!", in Michael Krausz, ed., *Relativism: interpretation and confrontation,* Notre Dame: University of Notre Dame Press.

[12] Kartik, Navin, Marco Ottaviani, and Francesco Squintani (2007), "Credulity, lies, and costly talk", *Journal of Economic Theory* 134: 93–116.

[13] Kandori, Michihiro (1992), “Social norms and community enforcement”, *Review of Economic Studies*, 59, 63–80.

[14] Mas-Collel, Andreu, Michael Whinston, and Jerry Green. (1995) *Microeconomic Theory,* Oxford: Oxford University Press.

[15] Mitchell, William C. (1984), “Schumpeter and Public Choice, part I. Precursor to Public Choice?”, *Public Choice* 42: 73-88.

[16] North, Douglass (1981), *Structure and change in economic history*, New York, NY: Norton & Co.

[17] Rescorla, Michael (2010) “Convention”, Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/entries/convention/>.

[18] Schelling, Thomas C. (1960), *The Strategy of Conflict*, Harvard, MA: Harvard University Press.

[19] Schumpeter, Joseph A. (1942), *Capitalism, socialism and democracy*, New York, NY: Harper and Brothers.

[20] Smith, Adam (1759), *The theory of moral sentiments,* London: Andrew Millar.

[21] Smith, Adam (1776), *An inquiry into the nature and causes of the wealth of nations,* London: W. Strahan and T. Cadell.

[22] Spence, Michael (1973), “Job market signaling”, *Quarterly Journal of Economics* 87: 355–374.

[23] Stein, Ernesto H., and Jorge M. Streb (2004), “Elections and the timing of devaluations”, *Journal of International Economics* 63: 119–145.

[24] Streb, Jorge M., and Gustavo F. Torrens (2011), “Meaningful talk”, Documento de Trabajo 443, Universidad del Cema.

[25] Wang, Hefei (2009), “Reputation acquisition of underwriter analysts - Theory and evidence”, *Journal of Applied Economics* 12: 331–363.

Appendix

In this Appendix we present the proofs of Propositions 1 to 6.

A1 Proof of Proposition 1

Through a systematic use of the definition of Perfect Bayesian Equilibrium, it is tedious, but not difficult, to verify the following:

1. If $\alpha \leq \bar{\alpha}_1$ (equivalently $q \geq \bar{q}_1$), then:

   (i) The pooling equilibria on "θ_H" are given by (the pooling equilibria on "θ_L" are analogous):

   - Seller's messages: $m_S(\theta_L)(\text{“}\theta_H\text{”}) = m_S(\theta_H)(\text{“}\theta_H\text{”}) = 1$;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) \in [0, \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L})$, $\mu(\text{“}\theta_H\text{”})(\theta_H) = q$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = q$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = \theta_L$, and $p_B(\text{“}\theta_H\text{”}) = \mathbf{E}_q[\theta]$.

   (ii) The hybrid equilibria are given by:

   - Seller's messages: $m_S(\theta_L)(\text{“}\theta_L\text{”}) = m_S(\theta_H)(\text{“}\theta_L\text{”}) = \sigma \in (0, 1)$;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) = \mu(\text{“}\theta_H\text{”})(\theta_H) = q$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = q$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = q$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = p_B(\text{“}\theta_H\text{”}) = \mathbf{E}_q[\theta]$.

2. If $\alpha > \bar{\alpha}_1$ (equivalently $q < \bar{q}_1$), then:

   (i) The pooling equilibria on "θ_H" are given by (the pooling equilibria on "θ_L" are analogous):

   - Seller's messages: $m_S(\theta_L)(\text{“}\theta_H\text{”}) = m_S(\theta_H)(\text{“}\theta_H\text{”}) = 1$;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) \in [0, \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L})$, $\mu(\text{“}\theta_H\text{”})(\theta_H) = q$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = q$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = p_B(\text{“}\theta_H\text{”}) = \theta_L$.

   (ii) The hybrid equilibria are given by:

   - Seller's messages: $m_S(\theta_L)(\text{“}\theta_L\text{”}) = \sigma_L$, $m_S(\theta_H)(\text{“}\theta_L\text{”}) = \sigma_H$, for σ_L, $\sigma_H \in (0, 1)$ such that the buyer's conjectures lead to expect $q < \bar{q}_1$ for either message;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) = \frac{\sigma_H q}{\sigma_L(1-q) + \sigma_H q} \in [0, \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L})$ and $\mu(\text{“}\theta_H\text{”})(\theta_H) = \frac{(1-\sigma_H)q}{(1-\sigma_L)(1-q) + (1-\sigma_H)q} \in [0, \frac{\alpha\theta_H - \theta_L}{\theta_H - \theta_L})$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = \mu(\text{“}\theta_H\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = p_B(\text{“}\theta_H\text{”}) = \theta_L$. ■

A2 Proof of Proposition 2

Through a systematic use of the definition of Perfect Bayesian Equilibrium, it is tedious, but not difficult, to verify the following:

1. If $\alpha \leq \bar{\alpha}_2$, then:

    (i) The pooling equilibria on "$\tilde{\theta}$" are given by:

    - Seller's messages: $m_S(\theta)\left(\text{“}\tilde{\theta}\text{”}\right) = 1$ for all $\theta \in [\theta_L, \theta_H]$;
    - Buyer's conjecture: $\mu\left(\text{“}\tilde{\theta}\text{”}\right)(\theta) = \frac{1}{\theta_H - \theta_L}$ (formally, this is the conditional density of θ, conditioning on the message "$\tilde{\theta}$"), while $\mu(\text{“}\theta\text{”})(\theta)$ for "θ" $\neq$ "$\tilde{\theta}$" is such that all type of sellers prefer to send "$\tilde{\theta}$";
    - Price offer: $p_B(\text{“}m\text{”}) = \mathbf{E}[\theta]$ for all "m".

    (ii) The hybrid equilibria are given by:

    - Seller's messages: $m_S(\theta)(\text{“}m\text{”}) = \sigma(\text{“}m\text{”}) \in [0, 1)$ for all θ.
    - Buyer's conjecture: if $m_S(\theta)(\text{“}m\text{”}) > 0$, then $\mu(\text{“}m\text{”})(\theta) = \frac{1}{\theta_H - \theta_L}$, if $m_S(\theta)(\text{“}m\text{”}) = 0$, then $\mu(\text{“}m\text{”})(\theta)$ such that all type of sellers prefer not to send that message;
    - Price offer: $p_B(\text{“}m\text{”}) = \mathbf{E}[\theta]$ for all "m".

2. If $\alpha > \bar{\alpha}_2$, then:

    (i) The pooling equilibria on "$\tilde{\theta}$" are given by:

    - Seller's messages: $m_S(\theta)\left(\text{“}\tilde{\theta}\text{”}\right) = 1$ for all $\theta \in [\theta_L, \theta_H]$;
    - Buyer's conjecture: $\mu\left(\text{“}\tilde{\theta}\text{”}\right)(\theta) = \frac{1}{\theta_H - \theta_L}$ (formally, this is the conditional density of θ, conditioning on the message "$\tilde{\theta}$"), while $\mu(\text{“}\theta\text{”})(\theta)$ for "θ" $\neq$ "$\tilde{\theta}$" is such that all type of sellers prefer to send "$\tilde{\theta}$";
    - Price offer: $p_B(\text{“}m\text{”}) = \frac{\alpha\theta_L}{2\alpha - 1}$ for all "m".

    (ii) The hybrid equilibria are given by:

    - Seller's messages: $m_S(\theta)(\text{“}m\text{”}) = \sigma(\text{“}m\text{”}) \in [0, 1)$ for all θ.
    - Buyer's conjecture: if $m_S(\theta)(\text{“}m\text{”}) > 0$, then $\mu(\text{“}m\text{”})(\theta) = \frac{1}{\theta_H - \theta_L}$, if $m_S(\theta)(\text{“}m\text{”}) = 0$, then $\mu(\text{“}m\text{”})(\theta)$ such that all type of sellers prefer not to send that message;
    - Price offer: $p_B(\text{“}m\text{”}) = \frac{\alpha\theta_L}{2\alpha - 1}$ for all "m". ■

A3 Proof of Proposition 3

By definition, honest sellers always reports the truth, i.e., $m_S(\theta_L, \chi)(\text{“}\theta_L\text{”}) = 1$ and $m_S(\theta_H, \chi)(\text{“}\theta_H\text{”}) = 1$. As we have shown in Section 4.1, $\mathbf{E}_{q,r}[\theta] \geq \alpha\theta_H$ if and only if $\alpha \leq \bar{\alpha}_3$.

On the one hand, suppose that $\alpha \leq \bar{\alpha}_3$. If all dishonest sellers reports "θ_L", i.e., $m_S(\theta_L, 0)(\text{“}\theta_H\text{”}) = m_S(\theta_H, 0)(\text{“}\theta_H\text{”}) = 1$, buyers are willing to pay $p_B(\text{“}\theta_L\text{”}) = \theta_L$ and $p_B(\text{“}\theta_H\text{”}) = \mathbf{E}_{q,r}[\theta]$. Given these price offers, sellers are not willing to change their messages, which implies that this is an equilibrium. Moreover, when $\alpha \leq \bar{\alpha}_3$, no other equilibrium exists since, if $m_S(\theta_L, 0)(\text{“}\theta_H\text{”}) \in [0, 1)$ and

$m_S(\theta_H, 0)(\text{“}\theta_H\text{”}) \in [0,1]$, it will always be the case that $p_B(\text{“}\theta_H\text{”}) > p_B(\text{“}\theta_L\text{”})$ and, hence, dishonest sellers with a lemon will prefer to lie, contradicting that $m_S(\theta_L, 0)(\text{“}\theta_H\text{”}) \in [0,1)$.

On other hand, when $\alpha > \bar{\alpha}_3$, the price $\mathbf{E}_{q,r}[\theta]$ buyers are willing to pay is not enough to make high quality sellers accept the offer. Thus, buyers will always pay $p_B(\text{“}\theta_L\text{”}) = p_B(\text{“}\theta_H\text{”}) = \theta_L$. This implies that dishonest sellers might be willing to randomize their messages.

Summing up, we have:

1. If $\alpha \leq \bar{\alpha}_3$ (equivalently $q \geq \bar{q}_3$), then the unique equilibrium is given by:
   - Seller's messages: $m_S(\theta_L, \chi)(\text{“}\theta_L\text{”}) = 1$, $m_S(\theta_L, 0)(\text{“}\theta_H\text{”}) = m_S(\theta_H, 0)(\text{“}\theta_H\text{”}) = m_S(\theta_H, \chi)(\text{“}\theta_H\text{”}) = 1$;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) = 0$, $\mu(\text{“}\theta_H\text{”})(\theta_H) = \frac{q}{(1-q)r+q}$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ for all p, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = \frac{q}{(1-q)r+q}$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ for $p < \alpha\theta_H$;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = \theta_L$, and $p_B(\text{“}\theta_H\text{”}) = \mathbf{E}_{q,r}[\theta]$.

2. If $\alpha > \bar{\alpha}_3$ (equivalently $q < \bar{q}_3$), then the set of equilibria are given by:
   - Seller's messages: $m_S(\theta_L, \chi)(\text{“}\theta_L\text{”}) = 1$, $m_S(\theta_H, \chi)(\text{“}\theta_H\text{”}) = 1$, $m_S(\theta_L, 0)(\text{“}\theta_L\text{”}) = \sigma_L$, $m_S(\theta_H, 0)(\text{“}\theta_L\text{”}) = \sigma_H$, for σ_L, $\sigma_H \in [0,1]$ such that;
   - Buyer's conjecture: $\mu(\text{“}\theta_L\text{”})(\theta_H) = \frac{\sigma_H qr}{\sigma_L(1-q)r+\sigma_H qr+(1-q)(1-r)} \in [0, \frac{\alpha\theta_H-\theta_L}{\theta_H-\theta_L})$ and $\mu(\text{“}\theta_H\text{”})(\theta_H) = \frac{(1-\sigma_H)qr+q(1-r)}{(1-\sigma_L)(1-q)r+(1-\sigma_H)qr+q(1-r)} \in [0, \frac{\alpha\theta_H-\theta_L}{\theta_H-\theta_L})$; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = \mu(\text{“}\theta_H\text{”})(\theta_H)$ for $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = p_B(\text{“}\theta_H\text{”}) = \theta_L$. ■

A4 Proof of Proposition 4

From Section 4.2, the equilibrium conditions are given by: (i) $p^* \geq \theta^*$; and (ii) $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$.

Let's begin computing $\mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$, under the assumption that $p^* \geq \alpha\theta_H$:

$$\mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*] = \frac{r\int_{\theta_L}^{\theta^*} \frac{\theta}{\theta_H-\theta_L} d\theta + \int_{\theta^*}^{\theta_H} \frac{\theta}{\theta_H-\theta_L} d\theta}{\Pr(\text{“}m\text{”} > \text{“}\theta^*\text{”})} = \frac{1}{2}\frac{(\theta_H)^2 - r(\theta_L)^2 - (1-r)(\theta^*)^2}{\theta_H - r\theta_L - (1-r)\theta^*}.$$

Though only $p^* \geq \theta^*$ is required, let $p^* = \theta^*$ (then we prove that $p^* = \theta^*$ maximizes expected quality, and makes the satisfaction of condition $p^* \geq \alpha\theta_H$ most likely). Substituting this, $p^* = \mathbf{E}[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*]$ leads to the following quadratic equation,

$$(p^*)^2 - \frac{2(\theta_H - r\theta_L)}{1-r}p^* + \frac{(\theta_H)^2 - r(\theta_L)^2}{1-r} = 0,$$

which has the roots $\frac{p^*}{\theta_H} = \frac{1-\sqrt{r}\left(\frac{\theta_L}{\theta_H}\right)}{1-\sqrt{r}}$ and $\frac{p^*}{\theta_H} = \frac{1+\sqrt{r}\left(\frac{\theta_L}{\theta_H}\right)}{1+\sqrt{r}}$. We can discard the first root, since it is larger than one, which violates the highest possible price buyers are willing to pay. Hence, what is required is $\frac{p^*}{\theta_H} = \frac{1+\sqrt{r}\left(\frac{\theta_L}{\theta_H}\right)}{1+\sqrt{r}} \geq \alpha$.

If $\frac{1+\sqrt{r}\left(\frac{\theta_L}{\theta_H}\right)}{1+\sqrt{r}} < \alpha$, it is not possible to support a Pareto efficient allocation. However, there is a partially informative equilibrium. In order to prove this, we only need to repeat the previous argument excluding a suitable range of high quality products. In particular, if we only consider qualities in the interval $\left[\theta_L, \frac{\theta^*}{\alpha}\right]$, such that the highest quality actually traded $\left(\frac{\theta^*}{\alpha}\right)$ exactly satisfies $\frac{1+\sqrt{r}\left(\frac{\theta_L}{\theta^*/\alpha}\right)}{1+\sqrt{r}} = \alpha$, so $\theta^* = \frac{\alpha\sqrt{r}}{\alpha(1+\sqrt{r})-1}\theta_L$ (where, of course, $\theta^* < \alpha\theta_H$), we already proved that there exists a Pareto efficient equilibrium with $p^* = \theta^*$.

Summing up, we have:

1. If $\alpha \leq \bar{\alpha}_4$, then there is an equilibrium given by:
   - Seller's messages: $m_S(\theta, h) =$ "θ" if $\theta \in [\theta_L, \theta^*]$ and $h = \chi$, $m_S(\theta, h) =$ "θ'" $\in$ ("θ^*", "θ_H"] if $\theta \in [\theta_L, \theta^*]$ and $h = 0$, $m_S(\theta, h) =$ "θ" if $\theta \in (\theta^*, \theta_H]$, where $\theta^* = (\theta_H + \sqrt{r}\theta_L)(1+\sqrt{r})^{-1}$;
   - Buyer's conjecture: μ ("θ") $(\theta) = 1$ if "θ" $\in$ ["θ_L", "θ^*"], μ ("θ" $\in$ ("θ^*", "θ_H"]) $(\theta \in (\theta^*, \theta_H]) = \frac{\theta_H - \theta^*}{r(\theta^*-\theta_L)+(\theta_H-\theta^*)}$, μ ("θ" $\in$ ("θ^*", "θ_H"]) $(\theta \in [\theta_L, \theta^*]) = \frac{r(\theta^*-\theta_L)}{r(\theta^*-\theta_L)+(\theta_H-\theta^*)}$;
   - Price offer: p_B ("θ") $= \theta$ if "θ" $\in$ ["θ_L", "θ^*"], p_B ("θ") $= \theta^*$ if "θ" $\in$ ("θ^*", "θ_H"].
2. If $\alpha > \bar{\alpha}_4$, then there is an equilibrium given by:
   - Seller's messages: $m_S(\theta, h) =$ "θ" if $\theta \in [\theta_L, \theta^*]$ and $h = \chi$, $m_S(\theta, h) =$ "θ'" $\in$ ("θ^*", "θ_H"] if $\theta \in [\theta_L, \theta^*]$ and $h = 0$, $m_S(\theta, h) =$ "θ" if $\theta \in (\theta^*, \theta_H]$, where $\theta^* = \frac{\alpha\sqrt{r}}{\alpha(1+\sqrt{r})-1}\theta_L$;
   - Buyer's conjecture: μ ("θ") $(\theta) = 1$ if "θ" $\in$ ["θ_L", "θ^*"], μ ("θ" $\in$ ("θ^*", "θ_H"]) $(\theta \in (\theta^*, \theta_H]) = \frac{\theta_H - \theta^*}{r(\theta^*-\theta_L)+(\theta_H-\theta^*)}$, μ ("θ" $\in$ ("θ^*", "θ_H"]) $(\theta \in [\theta_L, \theta^*]) = \frac{r(\theta^*-\theta_L)}{r(\theta^*-\theta_L)+(\theta_H-\theta^*)}$;
   - Price offer: p_B ("θ") $= \theta$ if "θ" $\in$ ["θ_L", "θ^*"], p_B ("θ") $= \theta^*$ if "θ" $\in$ ("θ^*", "θ_H"].

Proof that the quality cutoff point θ^* that maximizes expected quality in the market for regular goods is $\theta^* = p^*$

The conditional expected quality for messages of normal quality, $\mathbf{E}[\theta \mid \text{"}m\text{"} \geq \text{"}\theta^*\text{"}]$, reaches a maximum with respect to the quality cutoff point θ^* when it intersects the 45 degree line. To prove this, note that

$$\frac{\partial \mathbf{E}[\theta \mid \text{"}m\text{"} \geq \text{"}\theta^*\text{"}]}{\partial \theta^*} = \frac{-(\theta_H - r\theta_L)2\theta^* + (1-r)(\theta^*)^2 + (\theta_H)^2 - r(\theta_L)^2}{[\theta_H - r\theta_L - (1-r)\theta^*]^2}$$

Therefore $\frac{\partial \mathbf{E}[\theta \mid \text{"}m\text{"} \geq \text{"}\theta^*\text{"}]}{\partial \theta^*} > 0$ if and only if $(1-r)(\theta^*)^2 - (\theta_H - r\theta_L)2\theta^* + (\theta_H)^2 - r(\theta_L)^2 > 0$. This is a quadratic expression with two roots $\frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}}$ and $\frac{\theta_H - \sqrt{r}\theta_L}{1-\sqrt{r}}$. The first root belong to the interval $[\theta_L, \theta_H]$, but the second is higher than θ_H. Moreover, the quadratic expression evaluated at θ_L is positive. Therefore, $(1-r)(\theta^*)^2 - (\theta_H - r\theta_L)2\theta^* + (\theta_H)^2 - r(\theta_L)^2 > 0$ for $\left[\theta_L, \frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}}\right)$, $(1-r)(\theta^*)^2 - (\theta_H - r\theta_L)2\theta^* + (\theta_H)^2 - r(\theta_L)^2 < 0$ for $\left(\frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}}, \theta_H\right]$ and $(1-r)(\theta^*)^2 - (\theta_H - r\theta_L)2\theta^* + (\theta_H)^2 - r(\theta_L)^2 = 0$ for $\theta^* = \frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}}$. Hence, $\mathbf{E}[\theta \mid \text{"}m\text{"} > \text{"}\theta^*\text{"}]$ has a unique maximum at $\theta^* = \frac{\theta_H + \sqrt{r}\theta_L}{1+\sqrt{r}}$.

When we evaluate $\mathbf{E}\left[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^{*}\text{”}\right]$ at $\theta^{*}=\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}$ we get:

$$\begin{aligned}
\mathbf{E}\left[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^{*}=\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}\text{”}\right] &= \frac{1}{2}\frac{(\theta_H)^2-r(\theta_L)^2-(1-r)(\theta^{*})^2}{\theta_H-r\theta_L-(1-r)\theta^{*}} \\
&= \frac{1}{2}\frac{(\theta_H)^2-r(\theta_L)^2-(1-r)\left(\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}\right)^2}{\theta_H-r\theta_L-(1-r)\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}} \\
&= \frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}.
\end{aligned}$$

Hence, $\mathbf{E}\left[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^{*}\text{”}\right]$ has a unique maximum at $\theta^{*}=\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}$, and at that point $\mathbf{E}\left[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^{*}=\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}\text{”}\right]=\frac{\theta_H+\sqrt{r}\theta_L}{1+\sqrt{r}}$. Thus, $p^{*}=\theta^{*}$ is the quality cutoff point that maximizes expected quality. In other words, though any $p^{*}\geq\theta^{*}$ may support an equilibrium, $p^{*}=\theta^{*}$ makes the satisfaction of the condition $p^{*}\geq\alpha\theta_H$ most likely. ■

A5 Proof of Proposition 5

From Section 5.1 the equilibrium conditions are given by: (i) $h^{*}=\min\{p^{*}-\theta_L,\chi\}$; and (ii) $p^{*}=\mathbf{E}_{\tilde{\mu}}\left[\theta \mid \text{”}\theta_H\text{”},p^{*}\right]$, where

$$\mathbf{E}_{\tilde{\mu}}\left[\theta \mid \text{”}\theta_H\text{”},p^{*}\right]=\left[\frac{(1-q)\left(\frac{h^{*}}{\chi}\right)}{a_S(\theta_H,p^{*})(a)\,q+(1-q)\left(\frac{h^{*}}{\chi}\right)}\right]\theta_L+\left[\frac{a_S(\theta_H,p^{*})(a)\,q}{a_S(\theta_H,p^{*})(a)\,q+(1-q)\left(\frac{h^{*}}{\chi}\right)}\right]\theta_H$$

There are two possible cases to consider.

Case 1: First, it can be the case that all sellers with a lemon prefer to lie, i.e. $p^{*}-\theta_L\geq\chi$. Then, from (i) and (ii) $h^{*}=\chi$ and $p^{*}=(1-q)\theta_L+q\theta_H$. For this to be an equilibrium $p^{*}\geq\alpha\theta_H$, which is true if and only if $q\geq\bar{q}_1=\frac{\alpha\theta_H-\theta_L}{\theta_H-\theta_L}$. Finally we must check that in fact $p^{*}-\theta_L\geq\chi$, which is true if and only if $q(\theta_H-\theta_L)\geq\chi$. Therefore, the equilibrium for which all sellers with a lemon prefer to lie is given by $h^{*}=\chi$ and $p^{*}=(1-q)\theta_L+q\theta_H$, provided that $q\geq\max\left\{\frac{\chi}{\theta_H-\theta_L},\frac{\alpha\theta_H-\theta_L}{\theta_H-\theta_L}\right\}$.

Case 2: Second, it can be the case that only some sellers with a lemon prefer to lie, i.e. $p^{*}-\theta_L<\chi$. Then, from (i) and (ii) $h^{*}=p^{*}-\theta_L$ and $(1-q)(p^{*}-\theta_L)^2+\chi q(p^{*}-\theta_H)=0$. The solution of this quadratic equation is $p^{*}=\theta_L+\frac{q\chi}{2(1-q)}\left[\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{\chi q}}-1\right]$ (the other root does not belong to $[\theta_L,\theta_H]$). For this to be an equilibrium $p^{*}\geq\alpha\theta_H$, which is true if and only if $q\geq\bar{q}_5=\frac{(\alpha\theta_H-\theta_L)^2}{(1-\alpha)\theta_H\chi+(\alpha\theta_H-\theta_L)^2}$. Finally, we must check that in fact $p^{*}-\theta_L<\chi$, which is true if and only if $q(\theta_H-\theta_L)<\chi$. Therefore, the equilibrium for which only some sellers with a lemon prefer to lie is given by $h^{*}=p^{*}-\theta_L$ and $p^{*}=\theta_L+\frac{q\chi}{2(1-q)}\left[\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{\chi q}}-1\right]$, provided that $\frac{(\alpha\theta_H-\theta_L)^2}{(1-\alpha)\theta_H\chi+(\alpha\theta_H-\theta_L)^2}\leq q<\frac{\chi}{\theta_H-\theta_L}$.

In any other case, it is impossible to find a pair (h^{*},p^{*}) that simultaneously satisfies (i) and (ii). An equilibrium where only quality θ_L is traded at price θ_L in a pure price strategy equilibrium is not possible: if there is no gain from lying, all sellers have an incentive to state their true quality, which in turn leads buyers to want to pay a high price for announcement "θ_H"; on the other hand, if buyers are willing to pay a high price for announcement "θ_H", owners of lemons will want to mimic that message.

Summing up, we have:

1. Suppose that $\chi \leq \alpha\theta_H - \theta_L$.

   (i) If $q \geq \bar{q}_1$, then the equilibrium is given by:
   - Seller's messages: $m_S(\theta_L, h)(\text{“}\theta_H\text{”}) = m_S(\theta_H, h)(\text{“}\theta_H\text{”}) = 1$ for all h;
   - Buyer's conjecture: $\mu(\text{“}\theta_H\text{”})(\theta_H) = q$, $\mu(\text{“}\theta_L\text{”})(\theta_H) < \bar{q}_1$; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = q$ if $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_L) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ if $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_L) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = \theta_L$, and $p_B(\text{“}\theta_H\text{”}) = \mathbf{E}_q[\theta]$.

   (ii) If $q < \bar{q}_1$, then there is no equilibrium in pure price strategies.

2. Suppose that $\chi > \alpha\theta_H - \theta_L$

   (i) If $q \geq \frac{\chi}{\theta_H - \theta_L}$, then the equilibrium is given by:
   - Seller's messages: $m_S(\theta_L, h)(\text{“}\theta_H\text{”}) = m_S(\theta_H, h)(\text{“}\theta_H\text{”}) = 1$ for all h;
   - Buyer's conjecture: $\mu(\text{“}\theta_H\text{”})(\theta_H) = q$, $\mu(\text{“}\theta_L\text{”})(\theta_H) < \bar{q}_1$; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = q$ if $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_L) = \mu(\text{“}\theta_L\text{”})(\theta_H)$ if $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_L) = 0$ otherwise;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = \theta_L$, and $p_B(\text{“}\theta_H\text{”}) = E_q[\theta]$.

   (ii) If $\bar{q}_5 \leq q \leq \frac{\chi}{\theta_H - \theta_L}$, then the unique equilibrium is given by:
   - Seller's messages: $m_S(\theta_L, h)(\text{“}\theta_H\text{”}) = 1$ if $h < h^*$, $m_S(\theta_L, h)(\text{“}\theta_H\text{”}) = 0$ if $h \geq h^*$; $m_S(\theta_H, h)(\text{“}\theta_H\text{”}) = 1$ for all h;
   - Buyer's conjecture: $\mu(\text{“}\theta_H\text{”})(\theta_H) = \frac{q}{q + (1-q)\left(\frac{h^*}{\chi}\right)}$, $\mu(\text{“}\theta_L\text{”})(\theta_H) = 0$; $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = \mu(\text{“}\theta_H\text{”})(\theta_H)$ if $p \geq \alpha\theta_H$, $\tilde{\mu}(\text{“}\theta_H\text{”}, p)(\theta_H) = 0$ otherwise; $\tilde{\mu}(\text{“}\theta_L\text{”}, p)(\theta_L) = 0$ for all p;
   - Price offer: $p_B(\text{“}\theta_L\text{”}) = \theta_L$, and $p_B(\text{“}\theta_H\text{”}) = \theta_L + \frac{q\chi}{2(1-q)}\left[\sqrt{1 + \frac{4(1-q)(\theta_H - \theta_L)}{\chi q}} - 1\right]$.

   (iii) If $q < \bar{q}_5$, then there is no equilibrium in pure price strategies. ■

Proof of Corollary 1

The deduction of $q_{\min}$ is immediate from Proposition 5. The deduction of $\alpha_{\max}$ requires some algebra. From Proposition 5 Part 1, when $\chi \leq q(\theta_H - \theta_L)$, there is a Pareto efficient equilibrium for $\alpha \in \left[\frac{\chi + \theta_L}{\theta_H}, \frac{(1-q)\theta_L + q\theta_H}{\theta_H}\right]$. From Proposition 5 Part 2 (i), when $\chi \leq q(\theta_H - \theta_L)$, there is a Pareto efficient equilibrium for $\alpha < \frac{\chi + \theta_L}{\theta_H}$. Thus, when $\chi \leq q(\theta_H - \theta_L)$, $\alpha_{\max} = \frac{(1-q)\theta_L + q\theta_H}{\theta_H}$. From Proposition 5 Part 2 (ii), when $\chi > q(\theta_H - \theta_L)$, there is a Pareto efficient equilibrium when $\bar{q}_5 \leq q$ and $\alpha < \frac{\chi + \theta_L}{\theta_H}$.

It is tedious but not difficult to prove that the first inequality is satisfied if and only if $\alpha \leq \frac{\theta_L}{\theta_H} + \frac{q\chi\left(\sqrt{1 + \frac{4(1-q)(\theta_H - \theta_L)}{q\chi}} - 1\right)}{2(1-q)\theta_H}$. In order to verify this, note that the inequality $\frac{(\alpha\theta_H - \theta_L)^2}{\theta_H(1-\alpha)\chi + (\alpha\theta_H - \theta_L)^2} \leq q$ is satisfied if and only if $(1-q)(\theta_H)^2\alpha^2+[q\chi-2(1-q)\theta_L]\theta_H\alpha+(1-q)(\theta_L)^2-q\theta_H\chi\leq 0$. This quadratic expression has two roots: $\frac{\theta_L}{\theta_H}+\frac{q\chi\left(\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{q\chi}}-1\right)}{2(1-q)\theta_H}$ and $\frac{\theta_L}{\theta_H}-\frac{q\chi\left(\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{q\chi}}+1\right)}{2(1-q)\theta_H}$. Moreover, this quadratic expression evaluated at $\frac{\theta_L}{\theta_H}$ is negative. Hence, $\frac{(\alpha\theta_H-\theta_L)^2}{\theta_H(1-\alpha)\chi+(\alpha\theta_H-\theta_L)^2}\leq q$ is satisfied if and only if $\alpha\leq\frac{\theta_L}{\theta_H}+\frac{q\chi\left(\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{q\chi}}-1\right)}{2(1-q)\theta_H}$ (recall that $\alpha\theta_H>\theta_L$ always holds.)

Moreover, $\frac{\theta_L}{\theta_H}+\frac{q\chi\left(\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{q\chi}}-1\right)}{2(1-q)\theta_H}<\frac{\chi+\theta_L}{\theta_H}$, which implies that when $\chi>q(\theta_H-\theta_L)$, $\alpha_{\max}=\frac{\theta_L}{\theta_H}+\frac{q\chi\left(\sqrt{1+\frac{4(1-q)(\theta_H-\theta_L)}{q\chi}}-1\right)}{2(1-q)\theta_H}$. ■

A6 Proof of Proposition 6

From Section 5.2 the equilibrium conditions are given by: (i) $h^*(\theta)=\min\{p^*-\theta,\chi\}$; and (ii) $p^*=\mathbf{E}[\theta\mid\text{“}m\text{”}>\text{“}\theta^*\text{”},p^*]$. First, we need to compute q_{MIS}, $\mathbf{E}_{MIS}$, and $\mathbf{E}[\theta\mid\text{“}m\text{”}>\text{“}\theta^*\text{”},p^*]$. There are two possible cases to consider.

Case 1: Assume that in equilibrium $p^*-\theta_L\geq\chi$. Given $p^*=\theta^*$, the proportion of sellers that misrepresent quality is:

$$\begin{aligned}q_{MIS}=\Pr(\theta\in[\theta_L,\theta^*],\text{“}m\text{”}>\text{“}\theta^*\text{”})&=\frac{1}{\theta_H-\theta_L}\int_{\theta_L}^{p^*-\chi}d\theta+\frac{1}{\theta_H-\theta_L}\int_{p^*-\chi}^{p^*}\frac{p^*-\theta}{\chi}d\theta\\&=\frac{p^*-\theta_L-\frac{\chi}{2}}{\theta_H-\theta_L},\end{aligned}$$

and the average quality of those that misrepresent is

$$\begin{aligned}\mathbf{E}_{MIS}=\mathbf{E}(\theta\mid\theta\in[\theta_L,\theta^*],\text{“}m\text{”}>\text{“}\theta^*\text{”})&=\frac{\frac{1}{\theta_H-\theta_L}\int_{\theta_L}^{p^*-\chi}\theta d\theta+\frac{1}{\theta_H-\theta_L}\int_{p^*-\chi}^{p^*}\frac{p^*-\theta}{\chi}\theta d\theta}{q_{MIS}}\\&=\frac{p^{*2}-\theta_L^2-p^*\chi+\frac{\chi^2}{3}}{2p^*-2\theta_L-\chi}.\end{aligned}$$

Therefore, the average quality of those sellers that claim “m” > “θ^*” is given by:

$$\begin{aligned}\mathbf{E}[\theta\mid\text{“}m\text{”}>\text{“}\theta^*\text{”},p^*]&=\frac{q_{MIS}\mathbf{E}_{MIS}+\Pr(\theta\in[\theta^*,\theta_H])\,\mathbf{E}(\theta\mid\theta\in[\theta^*,\theta_H])}{\Pr(\theta\in[\theta^*,\theta_H])+q_{MIS}}\\&=\frac{\theta_H^2-\theta_L^2-p^*\chi+\frac{\chi^2}{3}}{2\theta_H-2\theta_L-\chi}\end{aligned}$$

In equilibrium, $p^*=\mathbf{E}[\theta\mid\text{“}m\text{”}>\text{“}\theta^*\text{”},p^*]$ must hold, which implies $\frac{p^*}{\theta_H}=\frac{1}{2}\frac{\theta_H+\theta_L}{\theta_H}+\frac{1}{6}\frac{\theta_H}{\theta_H-\theta_L}\left(\frac{\chi}{\theta_H}\right)^2$. For this to indeed be an equilibrium, we need to verify that $\alpha\theta_H\leq p^*\leq\theta_H$, i.e., $\sqrt{\frac{6\alpha\theta_H-3(\theta_H+\theta_L)}{\theta_H-\theta_L}}\leq$

$\frac{\chi}{\theta_H-\theta_L} \leq \sqrt{3}$. The upper limit is always satisfied in case 1 because $p^*-\theta_L \geq \chi$ implies $\frac{\chi}{\theta_H-\theta_L} \leq \left(3-\sqrt{6}\right)$, which is smaller than $\sqrt{3}$ (for higher values of misrepresentation costs, we move to case 2). As to the lower limit, $\alpha \leq 3-\sqrt{6}+\left(\sqrt{6}-2\right)\frac{\theta_L}{\theta_H}$ is needed for this condition to be satisfied, otherwise the solution set is empty.

Summing up case 1, where $\frac{\chi}{\theta_H} \leq \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$, there exists a Pareto efficient equilibrium if $\alpha \leq 3-\sqrt{6}+(\sqrt{6}-2)\frac{\theta_L}{\theta_H}$ and $\sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha-1-\frac{\theta_L}{\theta_H}\right)} \leq \frac{\chi}{\theta_H}$.

Case 2: Assume that in equilibrium $p^*-\theta_L \leq \chi$. Given $p^*=\theta^*$, the proportion of sellers that misrepresent quality is:

$$q_{MIS} = \Pr\left(\theta \in [\theta_L, \theta^*], \text{“}m\text{”} > \text{“}\theta^*\text{”}\right) = \frac{1}{\theta_H-\theta_L}\int_{\theta_L}^{\theta^*}\frac{\theta^*-\theta}{\chi}d\theta = \frac{\left(\theta^*-\theta_L\right)^2}{2\chi(\theta_H-\theta_L)},$$

and the average quality of those that misrepresent

$$\begin{aligned}\mathbf{E}_{MIS} = \mathbf{E}\left(\theta \mid \theta \in [\theta_L, \theta^*], \text{“}m\text{”} > \text{“}\theta^*\text{”}\right) &= \frac{\frac{1}{\theta_H-\theta_L}\int_{\theta_L}^{\theta^*}\frac{\theta^*-\theta}{\chi}\theta d\theta}{q_{MIS}} \\ &= \frac{\theta^{*3}-3\theta^*\theta_L^2+2\theta_L^3}{3\left(\theta^*-\theta_L\right)^2}\end{aligned}$$

Therefore, the average quality of those sellers that claim $m > \theta^*$ is given by:

$$\begin{aligned}\mathbf{E}\left[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*\right] &= \frac{q_{MIS}\mathbf{E}_{MIS}+\Pr\left(\theta \in [\theta^*, \theta_H]\right)\mathbf{E}\left(\theta \mid \theta \in [\theta^*, \theta_H]\right)}{\Pr\left(\theta \in [\theta^*, \theta_H]\right)+q_{MIS}} \\ &= \frac{\theta^{*3}-3\theta^*\theta_L^2+2\theta_L^3+3\chi\left(\theta_H^2-\theta^{*2}\right)}{3\left(\theta^*-\theta_L\right)^2+6\chi\left(\theta_H-\theta^*\right)}\end{aligned}$$

In equilibrium, $p^* = \mathbf{E}\left[\theta \mid \text{“}m\text{”} > \text{“}\theta^*\text{”}, p^*\right]$ must hold, which implies that we have the following cubic equation:

$$Q\left(\frac{p^*}{\theta_H}\right) = \frac{2}{3}\frac{\theta_H}{\chi}\left(\frac{p^*}{\theta_H}\right)^3 - \left(1+2\frac{\theta_L}{\chi}\right)\left(\frac{p^*}{\theta_H}\right)^2 + 2\left(1+\frac{\left(\theta_L/\theta_H\right)^2}{\chi/\theta_H}\right)\frac{p^*}{\theta_H} - \left(1+\frac{2}{3}\frac{\left(\theta_L/\theta_H\right)^3}{\chi/\theta_H}\right) = 0.$$

For this to be an equilibrium, the condition $\alpha\theta_H \leq p^* \leq \min\{\chi+\theta_L, \theta_H\}$ must be satisfied. A way to do this is to show that Q has a unique real root in the interval $\left[\alpha, \min\left\{\frac{\chi+\theta_L}{\theta_H}, 1\right\}\right]$. Note that: (a) $Q'\left(\frac{p^*}{\theta_H}\right) = 2\frac{\theta_H}{\chi}\left(\frac{p^*}{\theta_H}-\frac{\theta_L}{\theta_H}\right)^2 + 2\left(1-\frac{p^*}{\theta_H}\right) > 0$ since $\frac{p^*}{\theta_H} \leq 1$; (b) $Q(\alpha) = \frac{2}{3}\frac{\theta_H}{\chi}\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3 - (\alpha-1)^2$, which implies that $Q(\alpha) \leq 0$ if and only if $\frac{\chi}{\theta_H} \geq \frac{2}{3}\frac{\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$; (c) $Q\left(\frac{\chi+\theta_L}{\theta_H}\right) = -\frac{1}{3}\left(\frac{\chi}{\theta_H}\right)^2 + 2\left(1-\frac{\theta_L}{\theta_H}\right)\frac{\chi}{\theta_H} - \left(1-\frac{\theta_L}{\theta_H}\right)^2$, which implies that $Q\left(\frac{\chi+\theta_L}{\theta_H}\right) \geq 0$ if and only if $\frac{\chi}{\theta_H} \in \left[(3-\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right), (3+\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right)\right]$; and (d) $Q(1) = \frac{2}{3}\frac{\theta_H}{\chi}\left(1-\frac{\theta_L}{\theta_H}\right)^3 > 0$. We have to consider two possible situations. If $\frac{\chi}{\theta_H} \leq 1-\frac{\theta_L}{\theta_H}$, then Q has a unique real root in the interval $\left[\alpha, \frac{\chi+\theta_L}{\theta_H}\right]$ if and only if $\frac{\chi}{\theta_H} \geq \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ and $\frac{\chi}{\theta_H} \in \left[(3-\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right), 1-\frac{\theta_L}{\theta_H}\right]$. The second condition is always satisfied in case 2, since $\frac{\chi}{\theta_H} = (3-\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right)$ is the boundary point between cases 1 and 2. On the other hand, if $\frac{\chi}{\theta_H} > 1 - \frac{\theta_L}{\theta_H}$, then Q has a unique real root in the interval $[\alpha, 1]$ if and only if $\frac{\chi}{\theta_H} \geq \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$.

Summing up case 2, where $\frac{\chi}{\theta_H} \geq (3-\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right)$, there exists a Pareto efficient equilibrium whenever $\frac{\chi}{\theta_H} \geq \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$. This condition is always satisfied for $\alpha \leq 3-\sqrt{6}+(\sqrt{6}-2)\frac{\theta_L}{\theta_H}$, since in that range $(3-\sqrt{6})\left(1-\frac{\theta_L}{\theta_H}\right) \geq \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$. In this equilibrium $\frac{p^*}{\theta_H}$ is determined by the unique real root of the polynomial Q in the interval $\left[\alpha, \min\left\{\frac{\chi+\theta_L}{\theta_H}, 1\right\}\right]$.

In any other case, i.e., if $\frac{\chi}{\theta_H} < \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha - 1 - \frac{\theta_L}{\theta_H}\right)}$ for $\alpha \leq 3-\sqrt{6}+\sqrt{6}-2)\frac{\theta_L}{\theta_H}$, or $\frac{\chi}{\theta_H} < \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ for $\alpha > 3-\sqrt{6}+(\sqrt{6}-2)\frac{\theta_L}{\theta_H}$, it is not possible to support a Pareto efficient allocation. Nevertheless, it is not difficult to make a simple adjustment to the previous argument in order to prove that there exists a partially informative equilibrium. Let $\bar{\theta}_H = \max\left\{\tilde{\theta}_H, \hat{\theta}_H\right\}$, where $\tilde{\theta}_H$ and $\hat{\theta}_H$ are implicitly given by the following expressions:

$$\frac{\chi}{\tilde{\theta}_H} = \sqrt{3\left(1-\frac{\theta_L}{\tilde{\theta}_H}\right)\left(2\alpha - 1 - \frac{\theta_L}{\tilde{\theta}_H}\right)}, \text{ which always satisfies constraint } \alpha \leq 3-\sqrt{6}+\sqrt{6}-2)\frac{\theta_L}{\tilde{\theta}_H},$$

and

$$\frac{\chi}{\hat{\theta}_H} = \frac{2}{3} \frac{\left(\alpha - \frac{\theta_L}{\hat{\theta}_H}\right)^3}{(1-\alpha)^2}, \text{ if it satisfies constraint } \alpha > 3-\sqrt{6}+\sqrt{6}-2)\frac{\theta_L}{\hat{\theta}_H}, \text{ else } \hat{\theta}_H = \theta_L.$$

If $\bar{\theta}_H = \tilde{\theta}_H$, then if we only consider qualities in the interval $\left[\theta_L, \tilde{\theta}_H\right]$, we have already proved that there exists a Pareto efficient equilibrium with $\frac{p^*}{\tilde{\theta}_H} = \frac{1}{6}\left(\frac{\chi}{\tilde{\theta}_H}\right)^2 + \frac{1}{2}$. Similarly, if $\bar{\theta}_H = \hat{\theta}_H$, then if we only consider qualities in the interval $\left[\theta_L, \hat{\theta}_H\right]$, we have already proved that there exists a Pareto efficient equilibrium with $\frac{p^*}{\hat{\theta}_H}$ given by the unique solution of $Q\left(\frac{p^*}{\hat{\theta}_H}\right) = 0$.

Summing up, we have:

1. Suppose that $\alpha \leq \bar{\alpha}_6$

   (i) If $\frac{\chi}{\theta_H} \geq \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha - 1 - \frac{\theta_L}{\theta_H}\right)}$, then there exists an equilibrium given by:

   - Seller's messages: $m_S(\theta, h) =$ "θ" if $\theta \in [\theta_L, \theta^*)$ and $h \geq h^*(\theta)$, $m_S(\theta, h) \in$ ["θ^*", "θ_H"] if $\theta \in [\theta_L, \theta^*)$ and $h < h^*(\theta)$, $m_S(\theta, h) =$ "θ" if $\theta \in [\theta^*, \theta_H]$, where $h^*(\theta) = \min\{p^* - \theta, \chi\}$;
   - Buyer's conjecture: μ ("θ") $(\theta) = 1$ if "θ" $\in$ ["θ_L", "θ^*"], μ ("θ" $\in$ ("θ^*", "θ_H"]) (θ) such that $\mathbf{E}[\theta \mid$ "m" $\geq$ "θ^*"$] = \frac{\theta_H^2 - \theta_L^2 - p^*\chi + \frac{\chi^2}{3}}{2\theta_H - 2\theta_L - \chi}$;

- Price offer: $p_B(\text{“}\theta\text{”}) = \theta$ if $\theta \in [\theta_L, \theta^*]$ and $p_B(m) = p^*$ if “m” $\in$ [“θ^*”, “θ_H”], where $\frac{p^*}{\theta_H} = \frac{1}{6}\left(\frac{\chi}{\theta_H}\right)^2 + \frac{1}{2} = \frac{\theta^*}{\theta_H}$.

(ii) If $\frac{\chi}{\theta_H} \geq \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$,then there exists an equilibrium given by:

- Seller's messages: $m_S(\theta, h) =$ “θ” if $\theta \in [\theta_L, \theta^*)$ and $h \geq h^*(\theta)$, $m_S(\theta, h) \in$ [“θ^*”, “θ_H”] if $\theta \in [\theta_L, \theta^*)$ and $h < h^*(\theta)$, $m_S(\theta, h) =$ “θ” if $\theta \in [\theta^*, \theta_H]$, where $h^*(\theta) = \min\{p^* - \theta, \chi\}$;
- Buyer's conjecture: $\mu(\text{“}\theta\text{”})(\theta) = 1$ if “θ” $\in$ [”θ_L”,”θ^*”], μ (“θ” $\in$ (“θ^*”, “θ_H”]) (θ) such that $\mathbf{E}[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^*\text{”}] = \frac{\theta^{*3} - 3\theta^*\theta_L^2 + 2\theta_L^3 + 3\chi\left(\theta_H^2 - \theta^{*2}\right)}{3(\theta^* - \theta_L)^2 + 6\chi(\theta_H - \theta^*)}$;
- Price offer: $p_B(\text{“}\theta\text{”}) = \theta$ if $\theta \in [\theta_L, \theta^*]$ and $p_B(m) = p^*$ if “m” $\in$ [“θ^*”, “θ_H”], where $\frac{p^*}{\theta_H} = \frac{\theta^*}{\theta_H}$ is the unique solution of $Q\left(\frac{p^*}{\theta_H}\right) = 0$.

2. Suppose that $\alpha > \bar{\alpha}_6$. If $\frac{\chi}{\theta_H} \geq \frac{2}{3}\frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$, there exists an equilibrium given by:

- Seller's messages: $m_S(\theta, h) =$ “θ” if $\theta \in [\theta_L, \theta^*)$ and $h \geq h^*(\theta)$, $m_S(\theta, h) \in$ [“θ^*”, “θ_H”] if $\theta \in [\theta_L, \theta^*)$ and $h < h^*(\theta)$, $m_S(\theta, h) =$ “θ” if $\theta \in [\theta^*, \theta_H]$, where $h^*(\theta) = p^* - \theta$;
- Buyer's conjecture: $\mu(\text{“}\theta\text{”})(\theta) = 1$ if “θ” $\in$ [”θ_L”,”θ^*”], μ (“θ” $\in$ (“θ^*”, “θ_H”]) (θ) such that $\mathbf{E}[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^*\text{”}] = \frac{\theta^{*3} - 3\theta^*\theta_L^2 + 2\theta_L^3 + 3\chi\left(\theta_H^2 - \theta^{*2}\right)}{3(\theta^* - \theta_L)^2 + 6\chi(\theta_H - \theta^*)}$;
- Price offer: $p_B(\text{“}\theta\text{”}) = \theta$ if $\theta \in [\theta_L, \theta^*]$ and $p_B(m) = p^*$ if “m” $\in$ [“θ^*”, “θ_H”], where $\frac{p^*}{\theta_H} = \frac{\theta^*}{\theta_H}$ is the unique solution of $Q\left(\frac{p^*}{\theta_H}\right) = 0$.

3. Suppose that $\frac{\chi}{\theta_H} < \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha - 1 - \frac{\theta_L}{\theta_H}\right)}$ for $\alpha \leq \bar{\alpha}_6$, or $\frac{\chi}{\theta_H} < \frac{2}{3}\frac{\left(\alpha - \frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$ for $\alpha \geq \bar{\alpha}_6$. Then, there exists an equilibrium where only qualities in the interval $[\theta^*, \bar{\theta}_H]$, $\bar{\theta}_H = \max\{\tilde{\theta}_H, \tilde{\theta}_H\}$ defined above are actually traded, which is given by:

- Seller's messages: $m_S(\theta, h) =$ “θ” if $\theta \in [\theta_L, \theta^*)$ and $h \geq h^*(\theta)$, $m_S(\theta, h) \in$ [“θ^*”, “θ_H”] if $\theta \in [\theta_L, \theta^*)$ and $h < h^*(\theta)$, $m_S(\theta, h) =$ “θ” if $\theta \in [\theta^*, \theta_H]$, where $h^*(\theta) = \min\{p^* - \theta, \chi\}$;
- Buyer's conjecture: $\mu(\text{“}\theta\text{”})(\theta) = 1$ if “θ” $\in$ [”θ_L”,”θ^*”], μ (“θ” $\in$ (“θ^*”, “θ_H”]) (θ) such that $\mathbf{E}[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^*\text{”}] = \frac{\theta_H^2 - \theta_L^2 - p^*\chi + \frac{\chi^2}{3}}{2\theta_H - 2\theta_L - \chi}$ if $\bar{\theta}_H = \tilde{\theta}_H$, or $\mathbf{E}[\theta \mid \text{“}m\text{”} \geq \text{“}\theta^*\text{”}] = \frac{\theta^{*3} - 3\theta^*\theta_L^2 + 2\theta_L^3 + 3\chi\left(\bar{\theta}_H^2 - \theta^{*2}\right)}{3(\theta^* - \theta_L)^2 + 6\chi\left(\bar{\theta}_H - \theta^*\right)}$ if $\bar{\theta}_H = \tilde{\theta}_H$;
- Price offer: $p_B(\text{“}\theta\text{”}) = \theta$ if $\theta \in [\theta_L, \theta^*]$ and $p_B(m) = p^*$ if “m” $\in$ [“θ^*”, “θ_H”], where $\frac{p^*}{\tilde{\theta}_H} = \frac{\theta^*}{\tilde{\theta}_H} = \frac{1}{6}\left(\frac{\chi}{\tilde{\theta}_H}\right)^2 + \frac{1}{2}$ if $\bar{\theta}_H = \tilde{\theta}_H$, or $\frac{p^*}{\hat{\theta}_H} = \frac{\theta^*}{\hat{\theta}_H}$ is the unique solution of $Q\left(\frac{p^*}{\hat{\theta}_H}\right) = 0$ if $\bar{\theta}_H = \hat{\theta}_H$. ■

Proof of Corollary 2

From Proposition 6 Part 1, the minimum $\frac{\chi}{\theta_H}$ required to support a Pareto efficient equilibrium for $\alpha \leq \bar{\alpha}_6$ is $\frac{\chi}{\theta_H} = \sqrt{3\left(1-\frac{\theta_L}{\theta_H}\right)\left(2\alpha-1-\frac{\theta_L}{\theta_H}\right)}$. Inverting this function we obtain $\alpha_{\max}$ for $0 < \frac{\chi}{\theta_H} \leq \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$. The function is invertible since it is strictly increasing in α. Similarly, from Proposition 6 Part 2, the minimum $\frac{\chi}{\theta_H}$ required to support a Pareto efficient equilibrium for $\alpha \geq \bar{\alpha}_6$ is $\frac{\chi}{\theta_H} = \frac{2}{3}\frac{\left(\alpha-\frac{\theta_L}{\theta_H}\right)^3}{(1-\alpha)^2}$. Inverting this function we obtain $\alpha_{\max}$ for $\frac{\chi}{\theta_H} \geq \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$. Again, the function is invertible because it is strictly increasing in α. In order to prove continuity, there must be no jump at $\left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$: this is satisfied because $\frac{\left(\frac{\chi}{\theta_H}\right)^2}{6\left(1-\frac{\theta_L}{\theta_H}\right)} + \frac{1}{2}\left(1+\frac{\theta_L}{\theta_H}\right)$ evaluated at $\left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$ equals $f^{-1}\left(\left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)\right)$. From a direct inspection of $\alpha_{\max}$, it is an increasing and strictly convex function of $\frac{\chi}{\theta_H}$ for $\frac{\chi}{\theta_H} < \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$. For $\frac{\chi}{\theta_H} \geq \left(3-\sqrt{6}\right)\left(1-\frac{\theta_L}{\theta_H}\right)$, $f'(\alpha) = 2\left(\frac{\alpha-\frac{\theta_L}{\theta_H}}{1-\alpha}\right)^2 + \frac{4}{3}\left(\frac{\alpha-\frac{\theta_L}{\theta_H}}{1-\alpha}\right)^3 > 0$ and $f''(\alpha) = 4\left(1-\frac{\theta_L}{\theta_H}\right)^2\left(\alpha-\frac{\theta_L}{\theta_H}\right)(1-\alpha)^{-3} > 0$. Therefore, $\left(f^{-1}\right)' = \frac{1}{f'} > 0$ and $\left(f^{-1}\right)'' = \frac{-f''}{(f')^3} < 0$, which implies that f^{-1} is increasing and strictly concave. ■