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AN AXIOMATIC TREATMENT OF ENLARGED SEPARATION PORTFOLIOS AND TREASURER'S PORTFOLIOS (WITH APPLICATIONS TO FINANCIAL SYNTHETICS)

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AN AXIOMATIC TREATMENT OF ENLARGED SEPARATION PORTFOLIOS AND TREASURERS' PORTFOLIOS

(WITH APPLICATIONS TO FINANCIAL SYNTHETICS)

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ABSTRACT

Expanded separation portfolios (S^e) and Treasurer's portfolios T(S^e) are a sect of themselves. They arise out of risk-free assets and risky portfolios like other mutual funds. But their distinctive features set them apart from the common lot. This paper puts forth, firstly, a down-to-earth axiomatic that allows a complete formalization of the class of S^e portfolios. Secondly, simple separation portfolios are featured and their differences with (S^e) are highlighted. Next, the category of T(S^e) will be defined and their main properties brought to light. Last of all, there will be an expansion on the building of financial synthetics by means of enlarged separation portfolios.

JEL codes: G10, G11, G24

Key Words: *enlarged separation portfolios, mutual funds, separation portfolios, treasurer's portfolios, financial synthetics, portfolio management.*

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INTRODUCTION

It was James Tobin in his seminal paper of 1958 that laid the groundwork for mutual funds made out of risk-free assets and risky portfolios. On this line of analysis and adding some restrictive assumptions like capital market equilibrium under limitless arbitrage, William Sharpe (1964) produced the consequential CAPM model. It must be stressed, however, that here we are to understand by CAPM at least two things: the Capital Market Line (CML) and the Security Market Line (SML), as it was forcefully stressed by Brennan (1999) in his already classical contribution[1].

After Tobin and Sharpe, but in keeping with their agenda, one strand of research was undertaken by others scholars who added insight and precision to the issue of portfolios made out of risk-free assets and risky assets. The common ground about all these portfolios lies in the fact that they fulfill the Separation Theorem which states, as Elton and Gruber (1997) remarked, that " if an investor has access to a riskless asset, the choice of the optimum portfolio of risky assets is unequivocal and independent of the investor's taste for expected return or variance". Under the earlier and constraining Tobin`s assumptions this can also be translated, following Brennan (1999), as the pattern of behavior carried out by investors that firstly choose an optimal portfolio of risky assets and, afterwards but separately, they choose their own cash/risky asset ratio. It is not surprising that such portfolios started to be labeled "separation portfolios".

Another strand of academic contributions made inroads into what could be called the "physical world of Finance" and made the subject of separation portfolios a very lively and useful one in the actual practice of capital markets. On this regard, it's worth noticing that some books on investments have devoted a whole chapter to

[1] There is a widespread habit that makes CAPM and SLM synonyms. Although innocuous in usage, it might be misplaced at the end of the day.

so-called Capital Allocation Line, the geometrical locus where those mutual funds might be found[2].

This paper grows out of the latter strand of contributions and deals with enlarged separation portfolios, quite the opposite of separation portfolios, going on a line of research opened by Apreda (2001a, 2001b, 2003). On the other hand, it introduces the Treasurer's Portfolio, which is a derivative of the enlarged separation portfolio already studied by Apreda (2005a, 2003). The structure of S^e portfolios consists of risk-free asset but as a risky portfolio the one to be considered will be a market index albeit not necessarily optimum. The departure from the conventional view rests on the almost lack of stringent assumptions except five down-to-earth ones, to the extent that this paper resorts in some places to the single-factor model only. Taking advantages of this approach, we are going to show how the category of S^e portfolios might be used as financial synthetics.

As regards methodological issues, the paper comes up with a concise axiomatic approach to enlarged separation portfolios and Treasurer's portfolios, grounded on a down-to-earth viewpoint. It is our contention that such approach lends coherence and unity to the subject matter. The underlying format for this axiomatic treatment lends from the standard that Professor Edmund Landau (2001) set in his path breaking monograph on the foundations of Mathematical Analysis.

In section 1 we discuss the five Assumptions that will be relevant for ensuing definitions and lemmas. Next, in section 2, enlarged separation portfolios are defined. It is for section 3 to focus on plain separation portfolios to highlight their extent and limitations. Section 4 deals with enlarged separation portfolios performing like financial synthetics, while section 5 introduces and enlarges upon the Treasurer's portfolio.

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[2] See, for instance, Bodie et al. textbook on investments, chapter 6 (2006).

1. ASSUMPTIONS

We will only need five assumptions that are grounded on down-to-earth requirements so that the portfolios we are going to deal with in this paper become fully operational within what should be called "the physical world of Finance".

A1 There are risk-free assets[3]*.*

A2 There are market-indexed portfolios.

A3 Any qualified investor[4] *can always choose a convenient investment horizon*

$$\boldsymbol{H = [\, t;\, T\,]}$$

that fits his own strategy.

*A4 In building up their portfolios, any qualified investor can purchase risk-free assets as well as borrowing from risk free-assets. Most of the time the lending risk-free rate, **R(F, lending)**, is different than the borrowing risk-free rate, **R(F, borrowing)**.*

A5 Any qualified investor may buy market-indexes spot and sell them forward, instead of setting up a portfolio that proxies the underlying one in the chosen index.

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[3] Bear in mind that a financial asset is risk-free when it holds that

$$\mathbf{E(\, R(F)\,) \; = \; R(F)}$$

And this is true if and only if

$$\mathbf{E(\, R(F)\,) \; - \; R(F) \; = 0}$$

Or, equivalently,

$$\boldsymbol{\sigma^{2}\, (\, F\,) \; = \; E\, [\, E(\, R(F)\,) \; - \; R(F)\,]^{2} \; = \; 0}$$

[4] The expression "qualified investor" will be duly defined in the third remark below.

Remarks on Assumptions[5]

i. As regards A1, the temporal structure of rates of return stemming from Treasury Strips actually supply big players in the market with a broad range of zero-coupon bonds to replicate risk-free assets. In real life, even time deposits issued by banks carry out such role properly.

ii. Assumption 2 is also a plausible one, since market-indexed portfolios are widely supplied in global capital markets.

iii. By “**qualified investors**”, in A3, we mean the movers and shakers of the market. That is to say, dealers, brokers, investment banks, institutional investors, Treasury offices of multinational companies, commercial banks, investment funds (a variegated sort of them, displaying matching size, scope and scale so as to play in the league). It goes without saying that they get the hang of their investment horizon.

iv. Assumption 4 predicates that even big performers face two rates when lending to or borrowing from manifold sources like overnight Federal funds, as well as Repos and Reverses on assets with investment grade or, directly, in the inter-dealers market (by shorting or short-selling)[6].

v. Lastly, A5 refers to financial engineering devices by which the basic swap of buying-selling an index, instead of buying-selling its underlying basket of financial assets, it brings down costs and also enhances bundling advantages as well[7]. ■

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[5] As from now, the symbol ■ will denote “end of remarks”.

[6] Sometimes, the assumption about two different rates is narrowed down to a setting in which both rates are equal. For instance, the Capital Market Line makes this simplification, although some authors go further into details whenever two rates are of necessity (on this subject, see Elton – Gruber (2006), chapter 5).

[7] Going long at date **t**, and short at date **T**, allows for the index **I** to get as a holding return

2. ENLARGED SEPARATION PORTFOLIOS

In this section, we put forth the notion of enlarged separation portfolios, which ultimately become suitable vehicles for the framing of basic, cheap, and easily tractable portfolios[8].

Definition 1

By an enlarged separation portfolio we mean a portfolio

$$S^e = < x^e_F \, ; \; x^e_M >$$

given at date t in the horizon H, subject to the condition

$$x^e_F + x^e_M \neq 1$$

where x^e_F ***stands for a proportion of the starting cash balance allocated to a risk-free asset, and*** x^e_M ***stands for a market index proportion.***

Remarks on Definition 1

i. If the starting cash position amounted to

$$\mathbf{W(t)}$$

then the portfolio manager would allocate part of it, let us say **W(F, t)** to the risk-free asset and **W(M, t)** to the chosen market portfolio so that it holds:

$$x^e_F = W(F, t) \; / \; W(t) \quad ; \quad x^e_M = W(M, t) \; / \; W(t)$$

$$1 + R(M) = I(T) \; / \; I(t)$$

[8] Definition 1 stems from Assumptions 1, 2, and 3. The earliest development of enlarged separation portfolios can be found in Apreda (2003).

In this sort of portfolios, the key point lies on the fulfillment of the boundary condition

$$x^e_F + x^e_M \neq 1$$

or, to put it in a slightly different way,

$$W(F, t) + W(M, t) \neq W(t)$$

which points to the fact that we do not use up the starting cash balance. Instead, we keep an idle slack whenever

$$W(F, t) + W(M, t) < W(t)$$

or actually overspend through borrowing from risk-free assets, when

$$W(F, t) + W(M, t) > W(t)$$

ii. Borrowing may take place under different settings:

a) Seldom could qualified investors pay the same rate as the one they get when lending.

b) In general, they are able to borrow at a risk-free asset rate that is a little higher than the rate at which they lend.

c) Frequently, however, the actual borrowing rate is higher than the borrowing risk-free rate and, therefore, conveys a risk adjustment. ■

3. SEPARATION PORTFOLIOS

Although we can regard the notion of an enlarged separation portfolio as analogous to that of a separation portfolio, this perception would be misleading.

Actually, an enlarged separation portfolio is quite the opposite of the so-called separation portfolio. Let us delve with this issue in more detail[9].

Definition 2

By a separation portfolio we mean a portfolio

$$S = < x_F ; \ x_M >$$

at date t in the horizon H, such that

$$x_F + x_M = 1$$

where x_F ***stands for a proportion of the starting cash balance allocated to a risk-free asset, and*** x_M ***stands for a market index proportion.***

Remarks on Definition 2

i. The point at issue seems that, by resorting to simple separation portfolios the starting cash balance is partitioned into both components. That is to say:

$$\mathbf{W(F, t) + W(M, t) = W(t)}$$

ii. Therefore, the difference between an enlarged separation portfolio and a separation portfolio lies on how we ultimately apportion the starting cash balance:

   a) when designing separation portfolios, **W(t)** must be allocated to the free-risk asset and the market portfolio, using up the starting level of wealth;

   b) moreover, enlarged separation portfolios require a distinctive allocation to **F** and **M**, regardless whether **W(t)** is being fully used up or not.

[9] Definition 2 owes foundation to Assumptions 1, 2, and 3.

iii. For the sake of example, let us contrast four portfolios, two of them enlarged, the other being two separation portfolios, in each case with and without leverage:

$$S^{e} = < x^{e}_{F} ; x^{e}_{M} > = < 0.35 ; 0.45 >$$

$$S^{e} = < x^{e}_{F} ; x^{e}_{M} > = < -0.40 ; 1.80 >$$

$$S = < x_{F} ; x_{M} > = < 0.35 ; 0.65 >$$

$$S = < x_{F} ; x_{M} > = < -0.40 ; 1.40 > \blacksquare$$

A plain but powerful statement gives a necessary and sufficient condition to completely feature the category of separation portfolios[10].

Lemma 1

S is a separation portfolio if and only if it fulfills

$$E[R(S)] = R(F) + (< E[R(M)] - R(F) > / \sigma(M)) \times \sigma(S)$$

Proof:

By definition 2, the point of departure will be an arbitrary separation portfolio

$$S = < x_{F} ; x_{M} > \quad ; \quad x_{F} + x_{M} = 1$$

to be chosen at date **t** in the horizon **H**.

[10] a) It is noteworthy that all lemmas in this paper do not require stylized economic or equilibrium assumptions whatsoever beyond the five "down-to-earth" assumptions stated at the outset, in section 1.

b) By the way, Lemma 1 brings about the familiar equation of the Capital Market Line (CML), albeit we derive the outcome not resorting to CML's assumptions.

Let us work out the variance of such a portfolio.

$$\sigma^2_S = (x_F)^2 . \sigma^2_F + 2 x_F . x_M . \sigma(F; M) + (x_M)^2 . \sigma^2_M$$

But the risk-free asset has zero variance whereas covariances between the risk-free asset and the market portfolio are also nil[11]. Hence,

$$\sigma^2_S = (x_M)^2 . \sigma^2_M$$

or, equivalently,

(1)

$$\sigma_S = (x_M) . \sigma_M$$

and this leads to

(2)

$$x_M = \sigma_S / \sigma_M$$

Now, we figure out the expected return of **S**:

(3)

$$E[R(S)] = x_F . R(F) + x_M . E[R(M)]$$

Taking advantage of (2), we substitute[12] $\mathbf{x_M}$ and $\mathbf{x_F}$ in (3),

(4)

$$E[R(S)] = (1 - \sigma_S / \sigma_M) . R(F) + (\sigma_S / \sigma_M) . E[R(M)]$$

---

c) Lemmas 1 and 2 profit from Assumptions 4 and 5.

[11] This follows from the definitions of risk-free asset (see footnote 3) and covariance. For the latter:

$$\sigma(F; M) = E[(R(F) - E[R(F)]) . (R(M) - E[R(M)])]$$

$$\sigma(F; M) = E[(0) . (R(M) - E[R(M)])] = 0$$

[12] At this step we use the boundary condition

$$x_F + x_M = 1$$

by which this lemma and the next one couldn't hold for an enlarged separation portfolio.

Rearranging (4), **S** ultimately fulfills

$$\mathbf{E[\,R(\,S\,)\,] \;=\; R(\,F\,) \;+\; (\; < \; E[\,R(\,M\,)\,] \;-\; R(\,F\,) > / \;\; \sigma(\,M\,)\;)\; \times \;\sigma(\,S\,)} \tag{5}$$

This has proved necessity. For sufficiency, it is enough to revert implications from bottom to top[13]. ■

Remarks on Lemma 1

i. The message this lemma conveys is the following: if something is a separation portfolio, it lies on the line depicted by (5); conversely, if something lies on (5), it will be a separation portfolio for certain.

ii. Lemma 1 supposes that we can lend to and borrowing from at the same risk-free rate. In actual practice, and profiting from Assumption 4, we have to cope with two distinctive rates:

$$\mathbf{R(F) \;=\; R(F,\, lending)}$$

$$\mathbf{R(F) \;=\; R(F,\, borrowing)}$$

iii. Moreover, the necessary and sufficient condition of Lemma 1 remains true for this new setting, although it seems sensible to slightly change the notation used for the expected return of a separation portfolio **S**, by denoting

$$\mathbf{E[\,R_L\,(\,S\,)\,]} \;\text{ and }\; \mathbf{E[\,R_B\,(\,S\,)\,]}$$

the expected returns of the portfolio **S** when the portfolio manager lends or borrows, respectively. ■

Lemma 2

[13] As from now, the symbol ■ will also denote "end of proof" (see footnote 5).

S is a separation portfolio if and only if it fulfills

(6)

$$E[R_L(S)] = R(F, lending) + (< E[R(M)] - R(F, lending) > / \sigma(M)) \times \sigma(S)$$

$$when \quad \sigma(S) < \sigma(M)$$

(7)

$$E[R_B(S)] = R(F, borrowing) + (< E[R(M)] - R(F, borrowing) > / \sigma(M)) \times \sigma(S)$$

$$when \quad \sigma(S) > \sigma(M)$$

Proof:

The outcome is brought about by two conditions, bjoth of which follow from relationship (1) in Lemma 1:

a) $\sigma(S) < \sigma(M) \Leftrightarrow x(F) < 1 \Leftrightarrow F = F(lending)$

b) $\sigma(S) > \sigma(M) \Leftrightarrow x(F) < 0 \Leftrightarrow F = F(borrowing)$

and, by using the proof in Lemma 1, it is for (6) and (7) to ensue. ■

Remarks on Lemma 2

i. A higher borrowing risk-free rate means that the intercept of (7) in the second equation is higher than the intercept of (6). Furthermore, risk-premiums fulfill

$$[E[R(M)] - R(F, lending)] \quad > \quad [E[R(M)] - R(F, borrowing)]$$

so the slope of (7) is lower than the slope of (6), which means that the expected return increases as risk does it, but to a lesser extent than when the lending risk-free asset is used instead.

ii. As we attempt to deal with the physical world of Finance, (6) goes through the risk-free asset location **[0; R(F(lending))]** and the market-indexed portfolio location **[σ(M); E(R(M))],** whereas (7) goes through the locations of the risk-free asset **[0; R(F(borrowing))]** and the market-indexed portfolio **[σ(M); E(R(M))].** Adding more stressing assumptions like those holding in the CLM world, an interesting discussion ensues that allow for an in-between piece of arc belonging to the Markowitz' efficient frontier[14]. ■

4. ENLARGED SEPARATION PORTFOLIOS AS FINANCIAL SYNTHETICS

This is a rather elusive concept that might be laid upon either from a cash-flow viewpoint (a customary practice in derivatives analysis) or from a chosen risk-return profile. The latter standpoint is the one we are going to follow here, and the chosen risk metrics will be the "beta coefficient". In contradistinction with conventional usage, the only assumptions we need in the paper are the ones underlying the so-called single factor model[15].

Definition 3

Let us imagine that we single out certain financial asset (or portfolio) A whose risk-return profile at date t is given by the vector

$$< \beta(A) ; E[R(A)] >$$

By a financial synthetic of A is meant another financial asset (or portfolio) Y that exhibits the same risk-return profile than A. That is to say,

- $\beta(Y) = \beta(A)$

[14] See Elton-Gruber (2006), chapter 5.

[15] The nature of this model is statistical rather than economic. In other words we do not assume, for instance, CAPM's stronger assumptions since the approach of this paper intends to formalize a down-to-earth setting in the "physical world of Finance".

- $E[R(Y)] = E[R(A)]$

In next lemma, we are to take advantage of a striking relationship between the beta coefficient and the proportion of market index. Besides, it holds for both enlarged separation and simple separation portfolios.

Lemma 3

Given any enlarged separation portfolio at date t in the horizon H

$$S^e = < x^e_F ;\ x^e_M >$$

or any simple separation portfolio

$$S = < x_F ;\ x_M >$$

it holds that

$$\beta(S^e) = x^e_M \quad ; \quad \beta(S) = x_M$$

Proof:

From the single-factor model[16], we know that for **N** financial assets $\mathbf{A_1, A_2, \dots, A_N}$, and any portfolio made out of those assets

$$\mathbf{P} = < x_1, x_2, \dots, x_N >$$

the beta of this portfolio comes defined as

$$\beta(P) = \sum x_k \cdot \beta(A_k) \tag{8}$$

Down to enlarged separation portfolios, (8) amounts to:

[16] See Elton-Gruber (2006), chapter 7.

$$\beta(S^e) = x^e_F \, . \, \beta(F) + x^e_M \, . \, \beta(M)$$

and, lastly[17],

$$\beta(S^e) = x^e_M$$

The outcome for plain separation portfolios follows a similar line of argument. ■

Remark on Lemma 3

We have to bear in mind that the shorter the investment horizon lasts, the safer the risk position opened on the synthetic becomes. ■

Two applications

For the sake of illustration, let us consider a financial asset **A** whose risk-return profile at date **t** is given by the vector:

$$< \beta(A) \; ; \; E[R(A)] > \; = \; < 1.40 \; ; \; 10\% >$$

What if, due to transaction costs or regulations, the asset would not be available? To cope with such scenario, we will work out two alternative procedures for building up a synthetic of **A:** firstly, by means of a portfolio made out of two assets[18]; secondly, with an enlarged separation portfolio.

- *Alternative 1* *The synthetic is a portfolio of two distinctive assets*

Let us imagine that we have in stock two financial assets whose risk-return profile are

$$< \beta(B) \; ; \; E[R(B)] > \; = \; < 0.90 \; ; \; 6\% >$$

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[17] This stems from

$$\beta(F) = cov(R(F); R(M)) / \sigma^2_M = 0 \text{ and, secondly, that}$$

$$\beta(M) = cov(R(M); R(M)) / \sigma^2_M = 1.$$

[18] In actual practice, we do not need to constrain ourselves to a pair of assets only.

$$\langle \beta(C) ; E[R(C)] \rangle = \langle 1.70 ; 14\% \rangle$$

The synthetic of **A** will stem from a portfolio **P** consisting of **B** and **C**, that is to say:

$$\mathbf{P = \langle x(B) ; x(C) \rangle} \quad \text{so that} \quad \mathbf{x(B) + x(C) = 1}$$

Matching definition 3 implies

$$\langle \beta(A) ; E[R(A)] \rangle = \langle \beta(P) ; E[R(P)] \rangle$$

that is equivalent to the following system of equations:

(9)

$$\text{i.} \quad x(B) \,.\, E[R(B)] + x(C) \,.\, E[R(C)] = E[R(A)] = 10\%$$

$$\text{ii.} \quad x(B) \,.\, \beta(B) + x(C) \,.\, \beta(C) = \beta(A) = 1.40$$

Plugging numbers into (9) and solving, we get:

$$x(B) = 1.2574 \; ; \quad x(C) = 0.1754$$

That is to say:

$$P = \langle x(B) ; x(C) \rangle = \langle 1.2574 ; 0.1754 \rangle$$

and it holds,

$$x(B) + x(C) \neq 1$$

- *Alternative 2* *The synthetic is an enlarged separation portfolio*

The shortcoming in the former alternative is that, in real practice, either we have not the assets in stock or the solving proportions are not safely met in practice. To go beyond these constraints, we take advantage of enlarged separation portfolios.

To synthesize **A**, we need an enlarged separation portfolio

$$S^{e} = < x^{e}_{F} ; \; x^{e}_{M} >$$

such that

$$< \beta(A) ; E[R(A)] > \; = \; < \beta(S^{e}) ; E[R(S^{e})] > \; = \; < 1.40 ; 10\% >$$

The fulfillment of these constraints, and taking into account that the risk-free asset beta is nil, leads to the following:

$$\beta(S^{e}) = x^{e}_{F} \, \beta(F^{e}) + x^{e}_{M} \, \beta(M) = x^{e}_{M} = 1.40$$

$$E[R(S^{e})] = x^{e}_{F} \, E[R(F)] + x^{e}_{M} \, E[R(M)] = 10\%$$

If the risk-free asset earned a return of 4% and the market return were assessed at 8% then:

$$x^{e}_{F} \, . \, 4\% + 1.40 \, . \, 8\% = 10\%$$

and the proportion of risk-free asset would be

$$x^{e}_{F} = -0.30$$

Therefore, the enlarged separation portfolio would be a leveraged one:

$$S^{e} = < x^{e}_{F} ; \; x^{e}_{M} > \; = \; < -0.30 ; \; 1.40 >$$

Remark on Alternative 2

From assumptions 1 through 5, $\mathbf{S}^{e}$ is technically feasible and available in the physical world of Finance. But a contention may arise as to what extent higher levels of risk are to be allowed. In point of fact, the financial engineering unit should not open a long position in the market index whose **σ(M)** may be located beyond a prudentially established threshold[19]. ■

5. THE TREASURER'S PORTFOLIO

Let us imagine that some financial engineer, who is working for a big player in the market, draws up a financial synthetic of certain asset that, for the time being, is not already affordable[20]. For the sake of illustration, we keep on profiting from the example developed in section 4, where we had set up the enlarged separation portfolio

$$\mathbf{S^{e} = < x^{e}_{F}\ ;\ \ x^{e}_{M} >}$$

$$\mathbf{S^{e} = < -0.30\ ;\ \ 1.40 >}$$

At reaching this point, when it is for the financial engineer to request from the Treasurer the purchase (or selling) of the synthetic, some key issues arise eventually:

a) Firstly, the design of portfolios by the financial engineering unit must be constrained to daily starting cash balances[21] that are budgeted and allocated by the Treasury Office.
b) If the portfolio $\mathbf{S^{e}}$ needs to overspend in the market-indexed portfolio and

[19] Usually, the Risk Committee should be accountable for the setting of this risk threshold.

[20] This might plausibly be due to higher transaction costs, impairing regulatory constraints, lack of supply, among the main reasons.

[21] Although it falls outside the scope of this paper, we must add that, for big players, this should be a good governance practice, since there are many business centers within their organizations in competition for daily resources. Therefore, to enforce starting cash balances becomes a disciplinary rule of the game. Each business center, moreover, will request slacks from the Treasurer's or, alternatively, will provide its own slack to the Treasurer [on the semantics of Corporate Governance, see Apreda (2007, 2005b, 2005c)].

$$x^{e}_{F} + x^{e}_{M} > 1$$

then the Treasurer should lend the financial engineering unit by borrowing cash balances at a cost that not always might be assimilated to a risk-free asset rate, **R(F, borrowing)**. Most of the time, a higher rate has to be paid. The proportion of wealth to match this setting will be denoted by

$$x_{slack}$$

and it follows that

$$x^{e}_{F} + x^{e}_{M} + x_{slack} = 1$$

since the Treasurer's position must be squared by necessity.

c) By the same token, if the portfolio $\mathbf{S}^{e}$ can be purchased or sold leaving a free cash flow remaining from the financial starting balance, that is to say,

$$x^{e}_{F} + x^{e}_{M} < 1$$

In this scenario, the financial engineering unit "lends" this time to the Treasurer who records his own transaction as fulfilling

$$x^{e}_{F} + x^{e}_{M} + x_{slack} = 1$$

d) Most of the time a maximum threshold level of risk will be enacted, beyond which the Treasurer should deny the financial engineering units the requested transaction[22].

Therefore, the Treasurer must design a truly distinctive portfolio brought to light by next definition.

[22] See remark on Alternative 2 and footnote 18.

Definition 4

By the Treasurer´s portfolio T in connection with certain enlarged separation portfolio given at date t in the horizon H

$$S^{e} = < x^{e}_{F} ;\ x^{e}_{M} >$$

it is meant the vector

$$T = T(S^{e}) = < x^{e}_{F} ;\ x^{e}_{M} ;\ x_{slack} >$$

such that

$$x^{e}_{F} + x^{e}_{M} + x_{slack} = 1$$

Remarks on Definition 4

i. It could be argued that the Treasurer's portfolio structure conveys a "Source and Application Statement" of cash flows.

ii. In our example, the Treasurer's portfolio will have the following structure:

$$\mathbf{T} = < x^{e}_{F} ;\ x^{e}_{M} ;\ x_{slack} >$$

$$\mathbf{T} = \mathbf{T}(S^{e}) = < -0.30 ;\ 1.40 ;\ -0.10 >$$

Besides,

$$x^{e}_{F} + x^{e}_{M} + x_{slack} = 1 \quad \blacksquare$$

The following lemma highlights some properties held by theTreasurer's portfolio.

Lemma 4

Let $T = T(S^{e})$ be the Treasurer's portfolio from S^{e} at date in the horizon H.

If

$$R(\text{ slack }) = R(\text{ F, lending }) = R(\text{ F, borrowing})$$

then it holds that

a) $\sigma_T = \sigma_{S^e}$

b) T brings about a simple separation portfolio S(T)

c) There is a commuting relationship among S^e***,*** $T(S^e)$ ***and S(T)***

Proof:

a) Let us choose any treasurer's portfolio

$$T = T(S^e) = < x^e_F \; ; \; x^e_M \; ; \; x_{slack} >$$

If we assume that

$$R(\text{ slack }) = R(\text{ F })$$

then it holds that[23]

$$\sigma^2_T = (x^e_F)^2 . \sigma^2_F + 2\, x^e_F . x^e_M . \sigma(F; M) + 2\, x^e_F . x_{slack} . \sigma(F; slack) +$$

$$+ 2\, x^e_M . x_{slack} . \sigma(M; slack) + (x^e_M)^2 . \sigma^2_M + (x_{slack})^2 . \sigma^2_{slack}$$

which leads to

$$\sigma^2_T = (x^e_M)^2 . \sigma^2_M$$

and, by means of (1):

$$\sigma_T = (x^e_M) . \sigma_M = \sigma_{S^e}$$

b) Let the following correspondence be defined, from the set of all treasurer's portfolios to the set of all simple portfolios :

$$\varphi : \text{set}(T) \subseteq R^3 \rightarrow \text{set}(S) \subseteq R^2$$

[23] See footnote 11.

such that

$$\varphi(T) = \varphi(T(S^e)) = \varphi(< x^e{}_F ;\; x^e{}_M ;\; x_{slack} >) =$$

$$\varphi(T) = < x^e{}_F + x_{slack} ;\; x^e{}_M >$$

Clearly,

$$x^e{}_F + x_{slack}$$

is the proportion of wealth allocated to the risk-free asset.

Besides, by definition 4

$$x^e{}_F + x^e{}_M + x_{slack} = 1$$

Therefore, $\varphi(T)$ is a separation portfoliio.

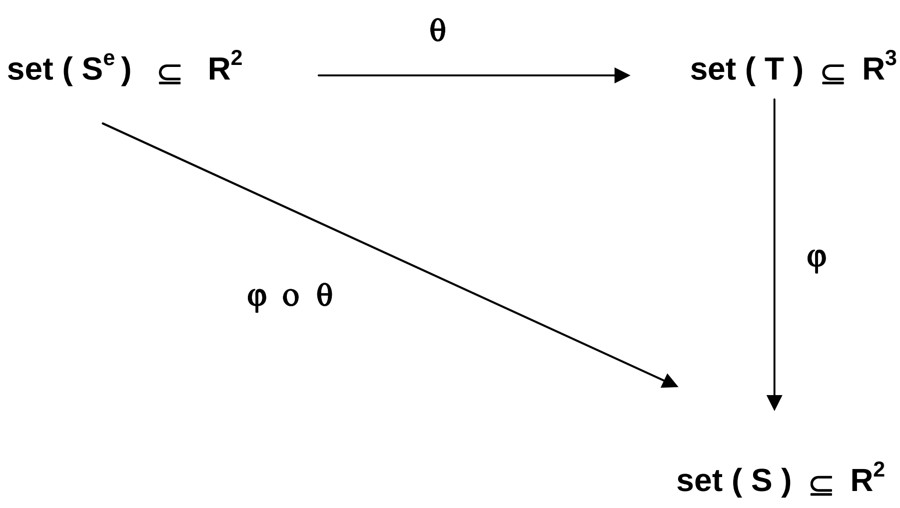
c) Let the following correspondence be defined, from the set of all enlarged separation portfolios to the set of all treasurer's portfolios:

$$\theta : \text{set}(S^e) \subseteq R^2 \rightarrow \text{set}(T) \subseteq R^3$$

such that

$$\theta(S^e) = T(S^e) = < x^e{}_F ;\; x^e{}_M ;\; x_{slack} >$$

Then, the following diagram is commutative:



that is to say:

$$\varphi \circ \theta(S^e) = < x^e{}_F + x_{slack} ; x^e{}_M >$$

$$\varphi [\theta(S^e)] = \varphi [T(S^e)] = < x^e{}_F + x_{slack} ; x^e{}_M >$$

We see that each **T** brings about a simple separation portfolio, whereas by the composition of both correspondences, each S^e brings about a simple portfolio. ■

Remarks on Lemma 4

Things are not so easy when the borrowing rate is not equal to the lending risk-free rate. In such case, two likely settings arise:

i. If **R(F, lending) < R(F, borrowing)**, then point a) in lemma 4 holds true, but b) and c) fail since we have quite another risk free asset in the third component of the Treasurer's portfolio.

ii. If **R(borrowing)** does not stem from a risk-free asset, then it holds that

$$\text{R(F, borrowing)} < \text{R(borrowing)}$$

and point a) in lemma 4 also fails since covariances between the borrowing rate and the lending rate are not equal to zero.

Corollary to Lemma 4

Within the context of Lemma 4, it follows that

$$\beta(T) = \beta(S^e)$$

Proof: By lemma 4,

$$\sigma_T = \sigma_{S^e}$$

whereas by lemma 3,

$$\beta(S^e) = x^e_M \tag{10}$$

On the other hand:

$$\beta(T) = x^e_F \, . \, \beta(F) + x^e_M \, . \, \beta(M) + x_{slack} \, . \, \beta(slack)$$

By assumption 4 on big players in the market, the slack refers either to **F (borrowing)** or **F(lending)**. Besides, betas of risk-free assets are null[24]. Hence:

$$\beta(T) = x^e_M \, . \, \beta(M) = x^e_M \tag{11}$$

By (10) and (11), it holds that

$$\beta(T) = \beta(S^e) \ \blacksquare$$

CONCLUSIONS

We have shown the extent to which enlarged separation and Treasurer's portfolios exhibit remarkable features in theory and practice:

a) They are feasible and cheap.
b) Their performance is easily tractable.
c) The category of $\mathbf{S^e}$ portfolios allows for the building up of financial synthetics straightfully.
d) As regards foundations, the paper laid up a comprehensive axiomatic treatment for the the categories of $\mathbf{S^e}$ and $\mathbf{T(S^e)}$ portfolios.

[24] See footnote 16.

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