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THE MARGINALIST REVOLUTION AND THE USE OF THE MATHEMATICAL
METHOD IN ECONOMICS

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*"I wish especially to become a good mathematician without
which nothing, I am convinced, can be thoroughly done."*

W. S. Jevons in a letter to her sister.

I. INTRODUCTION.

This paper studies the relation between the marginalist revolution and the use of the mathematical method in economics. A first version of this work was written as a term paper when I was a student of George Stigler at the University of Chicago. The present version has been benefitted from his comments and suggestions, as well as from the original bibliographical references provided by professor Stigler.

The paper is divided in four sections. Section 2 will propose the hypothesis that the mathematical method began to receive approval around 1889. Section 3 will establish the relation between the use of the mathematical method and the marginalist revolution. In Section 4 we will analyze why, if the marginalist revolution began in 1871, the mathematical method didn't receive much use or approval during the period 1871-1888; we also will discriminate between the positions of the founders of the

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marginalist school with respect to the use of mathematics in economics. Finally, Section 5 will summarize the main conclusions of the study.

II. 1889. A GOOD STARTING POINT.

We will propose in this section the hypothesis that the mathematical method began to be taken into account as a useful tool of analysis around 1889.

In order to support this hypothesis we will construct some statistics based on the bibliographies of mathematico-economic writings constructed by William Jevons (1931), and by Irving Fischer (1892).

Jevons constructed a list of 196 mathematico-economic writings, of these writings Irving Fischer selected those 50 that were undoubtedly mathematical, or closely associated with the mathematical method.

Fischer completed the Jevons list up to 1892 and made a more careful selection of the papers that were strictly and avowedly mathematical; making use of these papers he constructed an additional list of mathematico-economic papers. For example, in that list Fischer excluded Menger, Bohm-Bawerk and other writers of the Austrian school, who didn't use mathematical symbols.

Table A.1 (see the Appendix) presents a discrimination by group of years of the Fischer's selection from Jevon's list, of the Fischer's extensions of that list, and of the reduced Fischer's list of the strictly and avowedly mathematico-economic papers. Table A.2 (see the Appendix) presents a classification of the publications, by year, during the term 1870-1892.

Table 1 (see next page) reports the average of mathematico-economic publications, by year, during intervals of 5 years, since 1850. From this Table it becomes clear that the use of mathematics was sharply increased in the late 80's and early 90's.

TABLE 1

**YEARLY AVERAGE OF MATHEMATICO-ECONOMIC PUBLICATIONS,
DURING PERIODS OF 5 YEARS, SINCE 1850.**

<i>Period</i>	<i>Average per year</i>	<i>Average SAM</i>
	<i>(1)</i>	<i>(2)</i>
1850-1854	0.6	0.6
1855-1859	0.2	0.2
1860-1864	0.4	0.4
1865-1869	0.2	0.2
1870-1874	1.6	1
1875-1879	1.4	0.8
1880-1884	1.4	1
1885-1889	7.6	4.2
1890-1892	11.33	6

Source: Appendix, Table A.1.

where,

(1) = Constructed adding the items in Fischer's selection from Jevon's list to his own extension.

(2) = Includes only the items in the Fischer's list of strictly and avowedly mathematical (SAM) economic publications.

To specify the temporal location of the event it is interesting to construct a three years moving average since 1885 (see Table 2).

TABLE 2

**THREE YEARS MOVING AVERAGE OF THE NUMBER
OF MATHEMATICAL PUBLICATIONS**

<i>Year</i>	<i>Yearly MA</i>	<i>Yearly MA over SAM</i>
	<i>(*)</i>	<i>(**)</i>
1885	3	1.7
1886	3.7	2.7
1887	6	3.7
1888	10.7	5.7
1889	13.3	7.3
1890	13.3	6.7
1891	11.3	6

Source: Appendix, Table 2.

where,

(1) = Constructed as a three years moving average of the total number of mathematico-economic publications reported in Table A.2 of the Appendix.

(2) = Constructed as a three years moving average of the total number of strictly and avowedly mathematico-economic publications reported in Table A.2 of the Appendix.

The number of mathematico-economic publications raises clearly since 1888. During 23 years (1870-1892) the average of publications by year was 4.09 and of the strictly mathematical ones was 2.30. During 4 years (1889-1892) the yearly average was 13.25 and of the strictly mathematical one was 6.75. So, with this set of evidence, we will propose 1889 as a good starting point for the use of the mathematical method in economics.

III. THE USE OF MATHEMATICS IN ECONOMICS AND THE MARGINALIST REVOLUTION.

In this section we will investigate the existence of a causal relation from the marginalist revolution to the use of mathematics in economics.

Before 1838 the use of mathematics in economics was a rarity (see the Appendix, Table A.1). This fact is commonly reported in the literature; for example,

"The mathematical writers of the period before 1838 are listed, with a few exceptions, in the Jevons-Fischer bibliography of mathematico-economic literature.

The strong implication of this specialized bibliography - that the writings which are not

included contain little of interest - has proved to be correct."

Ross Robertson, 1949.

In 1838 an important fact must be remarked: Cournot's "Recherches sur les Principes Mathematiques de la Theorie des Richesses" was first published; this book was received with complete indifference by the public (for several years not a single copy was sold). Cournot is usually considered the founder of the math school (see, for example, Irving Fischer, 1898).

It is important to mention that the term mathematical school is used here not with the meaning to describe a group of writers who have a common program, but to denote those writers who used the same method. This fact is also mentioned by Robertson,

"Generally speaking pre-1838 attempt to use a mathematical method were made by those who adhered to no school, who, on the whole, took up economic investigation with few prejudgments and who frequently achieved amazing originality."

Ross Robertson, 1949.

The fact that the mathematical method was applied by adherents to no school was one important factor to explain why, in these early stages, it didn't attain a firm footing.

In 1863 Cournot rewrote the book without algebraic formulae calling it "Principes de la Theorie des Richesses," but neither this edition, nor a more elementary one "Revue Sommaire des Doctrines Economiques," published in 1876,

produced a different outcome. Gossen's book "Menschlichen Verkehrs," published in 1853, was equally unfortunate; up to Jevons nobody recognized its merits.

Therefore, as it is stated by Irving Fischer, before 1871 the use of the mathematical method in economics was a scientific curiosity,

"Has the mathematical method attained a firm footing? Before Jevons all the many attempt at mathematical treatment fell flat. Every writer suffered complete oblivion until Jevons unearthed their volumes in his bibliography....thus the mathematical method really began with Jevons in 1871"

Irving Fischer, 1892.

It is possible to infer that an important reason for this fact was that the classic theory had been clearly established, so it was not worth doing for most of the writers of the time to invest in mathematics to rewrite a theory that had been developed without the use of that tool.

This explanation suggests the causal relationship from the birth of the marginalist school to the use of mathematics in economics. In the rest of the section I will provide evidence in order support this hypothesis. A first step in direction to this goal will consist to analyze the opinions of some authors about the issue.

For example, Mark Blaug, 1962, suggests that,

"The dominant role of the concept of substitution at the margin in the new economics accounts for the sudden appearance of explicitly mathematical reasoning."

By the same token, James Guerity, 1965, states that,

"Although the development of the marginal concept and the introduction into economics of the techniques and modes of expression of formal mathematics do not coincide perfectly, there is enough coincidence to make their separate treatment extremely awkward and difficult."

Similarly, John Neville Keynes, 1904, asserts,

"It can, however, hardly be affirmed that there are economic truths of fundamental importance which are incapable of being expounded except in a mathematical form. Jevons' theory of utility and its applications are in many respects the most striking outcome of mathematical economics; and it is difficult to do full justice to this theory unassisted by mathematical methods."

Finally, it is interesting to quote Irving Fischer, 1892,

"It is perhaps fair to credit the idea of marginal utility to the mathematical method. The idea had five independent origins with Dupuit, Gossen, Jevons, Menger, and Walras. All except Menger presented this idea and presumably attained it by mathematical method. No idea has been more fruitful in the history of the science; this one achievement is a sufficient indication of the mathematical method."

From the analysis of this material it is fair to affirm that all these authors consider that exists a strong linkage between the use of mathematics in economics

and the marginalist revolution.

To give further evidence of this fact we will report opinions from, and about, some important marginalists.

For example, it is interesting to rewrite a small note from the ninth edition of the Encyclopedia Britannica,²

"L. Walras in Elements d'Economie Politique Pure (1874-1877) and Theorie Mathematique de la Richesse Sociale (1883), has followed the example of Cournot in attempting a mathematical treatment of the subject."

A last piece of evidence may be provided by the following quotation of one of the founders of the marginalist school, William Jevons, 1866, who explains that,

"The following paper briefly describes the nature of a theory of economy which will reduce the main problem of the science to a mathematical form. Economy, indeed being concerned with quantities has always of necessity been mathematical in its subject, but the strict and general statement and the easy comprehension of its quantitative laws has been prevented by a neglect of those powerful methods of expression which has been applied to most other sciences with so much success."

Therefore, it is possible to affirm that the marginalist revolution and the approval of the use of mathematics in economics are two facts that can be considered highly correlated; it is very difficult to figure the marginalist revolution without the use

²It is worth remarking that this edition was published in 1885.

of the new tool: the mathematics.

The marginal utility theory was introduced by Jevons, Menger and Walras. This is a matter of common knowledge, although the time at which the theory was effectively adopted comes later than common knowledge would have it. Why didn't the mathematical method receive much use or approval before 1889? Or in other words, what happened during the twenty years between 1871 and 1889? We will devote the following section to look for an answer to this question.

IV. THE DELAY ON THE ADOPTION OF THE MATHEMATICAL METHOD.

We have finished the third section of this paper with a question: why the mathematical method didn't receive much use or approval between 1871 and 1889? A first hint of an answer is suggested by the following quotation of Richard Howey, 1955,

"As said before, the present standard version of the history of the marginal utility school, which places the reader near 1870 in the company of Jevons, Menger and Walras, and directs him to look backward at the predecessors and forward to the successors, doubtless has as much, or more, in its favor as any of the alternatives mentioned above. "

but he added latter,

"If 1870 should prove unsatisfactory as a turning-point in the history of economic thought the alternative date of 1890 offers more attractions as a substitute than does

the earlier date proposed above (1854).... A history of the marginal utility school that shifts its principal date forward would start the history with the statement that around 1890, economists in Austria, England, the United States, and Switzerland began the widespread use of marginal utility in economic analysis that continues in some form in world economics down to the present day."

It is interesting to analyze this quotation together with the following quote of Irving Fischer, 1892,

"There has been a great increase in math literature since 1871. Just two decades have passed by since Jevons epoch-making book appeared. Of the mathematico-economic writings appearing in the period which here come to my notice, the number in the first decade was 30, representing 12 writers, while in the second decade it was 66, representing 23 writers. From all apparent evidence the mathematical method came to stay."

From the analysis of these statements it is possible to figure out that the mathematical method didn't receive much use or approval between 1871 and 1889 because the marginalist revolution took its time, 20 years, to obtain the generalized approval of the profession. In the following pages we will provide arguments in favor of this hypothesis.

When was the marginal utility theory adopted? In order to answer this question we require a method of characterizing the adoption of the theory by the economic science. George Stigler, 1982, provides that methodology by classifying economists

whether they did, or did not, understand the following proposition,

"The marginal utility of every commodity diminishes for every man, and this phenomenon underlies his demand curve for each commodity."

Table 3 summarizes the set of evidence generated by Stigler. The median date of first recognition of the utility theory by the economists listed was 1884, but several of the men had not begun to write on economics until later.

TABLE 3

**FIRST DATES OF RECOGNITION OF THE MARGINAL UTILITY
THEORY BY ECONOMISTS.**

<i>Date</i>	<i>Names</i>
1860-1869	Jevons.
1870-1879	Marshall, Cairnes, Walras.
1880-1884	Edgeworth, Sidgwick, Walker, Wicksteed, Wieser.
1885-1889	Bohm-Bawerk, Cannan, Auspitz and Lieben, Pantaleoni.
1890-1894	Fisher, Pareto, Taussig, Wicksell, Barone.

Source: Appendix, Table 3.

The delay in the adoption of the marginal utility theory is highlighted by Richard Howey, who proposes the hypothesis that the idea of marginal utility penetrated the

thinking of economists most slowly, and had to win its victories one by one, over inertia and even over active opposition. Howey asserts that the histories of economic thought often leave the impression that no sooner did Jevons, Menger and Walras publish, than their views informed the world and that accordingly, after 1871-1874 the thinking of economists had a different flavor than it had previously. He suggested that, by the contrary, the further history of the development of marginalism, from 1873 through most of the 1880's, was the history of a search for acceptance and support; acceptance that was finally obtained at the end of these twenty years.

In order to provide a different set of quantitative evidence it is interesting to rewrite the conclusions of part six of Howey's Ph. D. dissertation. He makes use of the number of references to marginal utility in the histories of economic thought during a given period as an indicator of the level of understanding of the idea. Table 4 synthesized Howey's conclusions.

TABLE 4

FREQUENCY OF APPEARANCE OF THE IDEA OF MARGINAL UTILITY IN THE HISTORIES OF ECONOMIC THOUGHT, AND THE RESULTING IMPLICATIONS.

<i>Period</i>	<i>Frequency</i>	<i>Conclusion</i>
1870-1889	No mention of marginal utility in the general histories of economic thought.	No widespread use of marginal utility as an analytical tool. Period of the rise of the marginal utility school.

1890-1909	Scattered and incomplete accounts of marginal utility found in general histories of economic thought.	Use of marginal utility by economists spreads rapidly. Period of the triumph of the marginal utility school.
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Source: Richard Howey, 1955.

By the time the theory was generally known to economists, the science was rapidly moving toward an academic character. Actually, as George Stigler reported, almost every economist who dealt seriously and professionally with the marginal utility theory had an academic base. If we date leading English, American, and French, economists by their mean year of publication, we find the academic participation to be rising during the period. This fact is reported in Table 5 (see next page).

It is interesting to mention, as a last piece of evidence that illustrates the interrelation between the marginalist school and the use of mathematics in economics by the writers of the time, the evaluation of John Ingram, 1907, about William Jevon's work.³ In his book, Ingram shows incredulity respect to the importance of Jevon's work. More than this, he considered that the expectation of being able to subject economic doctrine to a mathematical method will be found illusory.

³It must be taken into account that this History is considered by Richard Howey as the only one published in the period 1870-1889 which mentioned marginal utility. (The first edition of this book was published in 1888).

TABLE 5

PROMINENT ENGLISH, AMERICAN, AND FRENCH, ECONOMISTS, 1800-1915.

<i>Period</i>	<i>English</i>		<i>American</i>		<i>French</i>	
	<i>(1)</i>	<i>(2)</i>	<i>(1)</i>	<i>(2)</i>	<i>(1)</i>	<i>(2)</i>
1800-1825	nd	nd	nd	nd	4	2
1825-1850	14	0	nd	nd	7	3
1850-1875	10	3	4	3	8	5
1875-1900	7	2	14	7	2	2
1900-1915	8	5	23	21	5	3
1915 and after.	nd	nd	9	9	nd	nd

Source: Richard Howey, 1955.

where,

(1) = Total number of prominent economists.

(2) = Number of academic prominent economists.

nd = No data.

The analysis of the different sets of evidence provided supports the hypothesis that there is a causal relation from the marginalist revolution to the use of mathematics in economics and, as the marginalist revolution didn't obtain an instantly triumph in the 1870's, the adoption of the mathematical method neither was without a delay. The approval of the marginalist postulates was a slow process that finally

arrived to an end around 1890; this fact explains why during 20 years the mathematical method didn't receive much use or approval.

To finish this section it is important to remark a difference between the three founders of the marginalist school with relation to the use of mathematics in economics.

For example, Irving Fischer⁴ didn't include in the list of strictly and avowedly mathematical economic publications (see the Appendix, Table 1.A), those which were written by Menger; Bohm-Bawerk, and other writers of the Austrian school, who in spite of a mathematical tone have omitted the use of mathematical symbols,

"These three writers independently and almost simultaneously developed and applied the idea of rarity or marginal utility, the corner-stone of the so-called Austrian School and of the mathematical economics".

To provide another original illustration of the non-mathematical style of the Austrian School we will quote a description of this School that appeared in the Palgrave's Dictionary of Political Economy, published in 1910,

"Like Jevon's they find the key to the problem of value in the notion of final utility. Unlike Jevons they regard economics as having nearer affinity with psychology than with mathematics, and they are sparing in the use of mathematical illustration."

⁴This quotation of Irving Fischer comes from his introduction of "Geometrical Theory of the Determination of Prices," Leon Walras, 1892-1893.

A final illustration of the non-mathematical style of the Austrian School is provided by Emil Kauder, 1965; who compares the characteristics of the three founders of the marginalism. Table 6 summarizes his findings.

TABLE 6

JEVONS AND WALRAS VS. MENGER.

Walras, Jevons

Mathematic is especially suitable for solving problems, because it describes the relation of interdependent factors.

The laws of exchange can be expressed in mathematical equations.

Menger

Economic theory does not investigate the interdependence of economic phenomena but rather the essence of value, rent, profit, division of labor, bimetallism, etc.

Equations can lead only to arbitrary statements, not to exact laws. The description of an equilibrium can be only the end, not the beginning of an analysis.

Source: Emil Kauder, 1965.

Therefore, in order to highlight the existence of a causal relation from the marginalist revolution to the use of mathematics in economics we have to take into account the clear differences between the three founders of the marginalist school. The hypothesis that we have proposed refers obviously to Jevons, Walras and their

disciples, but not to the members of the Austrian school.

IV. CONCLUSIONS.

In this section we will list the main conclusions that we have reached:

1. The mathematical method began to receive approval around 1889.
2. There is a causal relation from the birth of the marginalist school to the use of mathematics in economics.
3. The mathematical method didn't receive much use or approval between 1871-1889 because the marginalist revolution took 20 years to obtain the generalized approval of the writers of its time.
4. It is necessary to differentiate the approach of the founders of the marginalist school with respect to the use of mathematics in economics. Jevons and Walras considered that mathematics was especially suitable for solving economic problems; this opinion was not shared by Menger, who thought that economics had a nearer affinity with psychology than with mathematics.
5. Given conclusion four, it is necessary to modify the second conclusion in the following way: there is a causal relationship from the birth of the marginalist school to the use of mathematics in economics. This causal relationship comes from the methodology utilized by Jevons, Walras and their disciples; but not from the writers of the Austrian school.

We want to finish this paper with a quotation of one of the founders of the

marginalist school: William Jevons⁵.

In a letter to his sister he expressed,

"I wish especially to become a good mathematician without which nothing, I am convinced, can be thoroughly done."

⁵This quotation can be found in Richard Howey, 1955.

APPENDIX

TABLE A.1

NUMBER OF PUBLICATIONS BY PERIOD.

<i>Period</i>	<i>Fischer Selection (2)</i>	<i>Extension by Fischer</i>	<i>Total</i>	<i>SAM (3)</i>
1700-1799	2		2	2
1800-1819	1		1	1
1820-1829	3		3	3
1830-1839	2		2	2
1840-1849	6		6	6
1850-1859	4		4	4
1860-1869	3	1	4	4
1870-1879	15		15	9
1880-1888 (1)	14	12	26	17
1889-1892		53	53	27

Source: Irving Fischer, 1892.

where,

(1) = Jevons list includes publications up to 1888.

(2) = Fischer selection from Jevons' list (50 out of 196).

(3) = Fischer list of strictly and avowedly mathematico-economic publications.

TABLE A.2

NUMBER OF PUBLICATIONS BY YEAR, DURING THE PERIOD 1870-1892.

<i>Period</i>	<i>Fischer Selection (1)</i>	<i>Extension by Fischer</i>	<i>Total</i>	<i>SAM (2)</i>
1870	1		1	1
1871	2		2	1
1872	1		1	1
1873	1		1	1
1874	3		3	1
1875	2		2	0
1876	3		3	3
1877	0		0	0
1878	1		1	1
1879	1		1	1
1880	0		0	0
1881	2		2	2
1882	0	1	1	1
1883	1		1	1
1884	1	2	3	1
1885	1		1	1
1886	5		5	3
1887	3	2	5	4
1888	1	7	8	4
1889		19	19	9
1890		13	13	9
1891		8	8	2
1892		13	13	7

Source: Irving Fischer, 1892.

where,

(1) = Fischer selection from Jevons' list (50 out of 196).

(2) = Fischer list of strictly and avowedly mathematico-economic publications.

TABLE A.3

FIRST DATES OF RECOGNITION OF THE MARGINAL UTILITY
THEORY BY ECONOMISTS.

<i>Name</i>	<i>Date</i>	<i>Place</i>
Jevons	1862	British Association for Advancement of Science
Menger	1871	Grundsätze der Volkswirtschaftslehre
Marshall	1872	Review of Jevons
Cairnes	1874	Leading Principles
Walras	1874	Elements d'économie
J. Clark	1881	Article in New Englander
Edgeworth	1881	Mathematical Psychics
Sidgwick	1883	Principles of Political Economy
Walker	1883	Principles
Wicksteed	1884	Review of Das Kapital in To-Day
Wieser	1884	Über den Ursprung des Wertes
Bohm-Bawerk	1886	Theorie des Guterwerts
Cannan	1888	Elementary Political Economy
Auspitz and Lieben	1889	Theorie des Preises
Pantaleoni	1889	Principii
Fisher	1892	Mathematical Investigations
Pareto	1892	Articles in Giornale degli Economisti
Taussig	1893	Proceedings, American Economic Association

TABLE A.3

FIRST DATES OF RECOGNITION OF THE MARGINAL UTILITY
THEORY BY ECONOMISTS.

<i>Name</i>	<i>Date</i>	<i>Place</i>
Wicksell	1893	Über Wert, Kapital und Rente
Barone	1894	Articles in Giornale degli Economisti
Cassel	1899	Articles in Zeitschrift für die gesamte Staatswissenschaft

Source: George Stigler, 1982.

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